A Study on Laplace Transformation and its Application

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Abstract: ***This research paper is a study of the Laplace transformation and its application. The researchers are interested in discussing the properties and applications of Laplace transform in various fields. The knowledge of Laplace transformation in recent years has been an essential part of the mathematical background required for engineers, physicists, scientists, and mathematicians. Laplace transform methods provide easy, simple, and effective means for the solutions of many problems arising in various fields of science, mechanics, engineering, and so on. With the increasing complexity of engineering problems, Laplace transforms help in solving complex problems with a very simple approach just like the applications of transfer functions to solve ordinary differential equations.***

The main objective of this research paper is a scientific review of the properties and applications of Laplace transform. This paper also includes the formulation of the Laplace transform of important functions like the periodic functions and the Unit Impulse function.

Keywords: ***Laplace transform, Properties, Differential equations.***

I. INTRODUCTION

This paper provides a brief explanation of what the Laplace Transform is and how it is used in daily life. A transformation is a mathematical operation that changes one mathematical expression into another's equivalent simple form. The important field of Mathematical Analysis is Laplace transformations, also known as integral transforms, which have numerous applications in several disciplines, including engineering technology, the fundamental sciences, mathematics, and economics. The majority of engineering problems are mathematically formulated as differential equations. We frequently use mathematical models and their applications in daily life. An essential role is played by the population growth model. The Laplace transformable us to solve differential equations by use of algebraic methods. Laplace transform is a mathematical tool that can be used to solve many problems in Science and engineering. This transform was first introduced by Laplace, a French mathematician, in the year 1790. The Laplace Transform is also used in solving differential and integral equations.

The Laplace Transform is a specific type of integral transform. Considering a function f (t), its corresponding Laplace Transform will be denoted as L[f(t)], where L is the operator operated on the time domain function f(t).

A. *Definition of Laplace Transform:* The Laplace transform of the function(t) for all $t\geq 0$ is defined as

$$[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where L is the Laplace transform operator. The Laplace transform of the function (t) for all $t\geq 0$ exists if (t) is in exponential order and piecewise continuous and if integral in (1) converges for some values of s, otherwise, it does not exist. These are only sufficient conditions for the existence of the Laplace transform of the function(t).

II. PROPERTIES OF LAPLACE TRANSFORM

1) Linearity Property

$$If[f(t)] = \bar{f}(s) \ \& \ L[g(t)] = \bar{g}(s) \text{ then}$$

$[af(t) + bg(t)] = aL[f(t)] + bL[g(t)]$ where a and b are arbitrary constants.

2) *First Shifting Property*

$$If\ L[f(t)] = \bar{f}(s), then\ L[e^{at}f(t)] = \bar{f}(s-a)$$

3) *Convolution Theorem*

$$If\ L^{-1}[\bar{f}(s)] = f(t)\ \&\ L^{-1}[\bar{g}(s)] = g(t)\ then$$

$$L^{-1}[\bar{f}(s) * \bar{g}(s)] = \int f(u)g(t-u)du$$

4) *Laplace transform of Derivative*

$$[f'(t)] = sL[f(t)] - f(0),$$

$$[f''(t)] = s^2L[f(t)] - sf(0)-f'(0),$$

$$L[f'''(t)] = s^3L[f(t)] - s^2f(0)-sf'(0)-f''(0), \text{ and so on.}$$

5) *Laplace transform of Integrals*

$$If\ L[f(t)] = \bar{f}(s), \quad then\ \ L\{\int_0^u f(u)du\} = \frac{1}{s}f(s)\ \ \&\ \int_0^t f(u)du = L^{-1}(\frac{f(s)}{s})$$

6) *Multiplication by t^n*

$$If\ L[f(t)] = \bar{f}(s), then \quad L\left(t^n f(t)\right) = (-1)^n \frac{d^n}{ds^n}\left(f(s)\right),\ n \in Z^+$$

7) *Division by t*

$$If\ L[f(t)] = \bar{f}(s), then \quad L\left(\frac{f(t)}{t}\right) = \int_s^\infty f(s)ds$$

8) *Laplace transform of Unit step function is*

$$L\left(u(t-a)\right) = \frac{1}{s}e^{-as} \dots\dots\dots where\ u(t-a) = \begin{cases} 0 & t<a \\ 1 & t\geq a, a\geq 0 \end{cases}$$

9) *Second Shifting Theorem*

$$If\ L[f(t)] = \bar{f}(s), then$$

$$L[f(t-a)u(t-a)] = e^{-as}\bar{f}(s)$$

10) *Laplace transform of unit Impulse function*

$$L[\delta(t-a)] = e^{-as}$$

$$where\ \delta(t-u) = \begin{cases} \infty & t=a \\ 0 & t\neq a \end{cases}$$

11) *Laplace transform of Periodic function*

$$L[f(t) = \frac{1}{1-e^{-st}}\int_0^T e^{-st} f(t)dt$$

III. LAPLACE TRANSFORM TABLE

F(s) = L{f(t)}	**F(t) = L^{-1}{F(s)}**
1	$\frac{1}{s}, s > 0$

t^n, n > 0	$\frac{n!}{s^{n+1}}, s > 0$
$t^n e^{at}$, n > 0	$\frac{n!}{(s-a)^{n+1}}, s > a$
e^{at}	$\frac{1}{s-a}, s > a$
sin(at)	$\frac{1}{s^2+a^2}, s > 0$
tsin(at)	$\frac{2as}{(s^2+a^2)^2}, s > a$
e^{at}sin(bt)	$\frac{b}{(s-a)^2+b^2}, s > a$
cos(at)	$\frac{s}{s^2+a^2}, s > 0$
tcos(at)	$\frac{s^2-a^2}{(s^2+a^2)^2}, s > a$
e^{at}cos(bt)	$\frac{s-a}{(s^2-a^2)^2+b}, s > a$
$f^n(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{n-1}(0)$

IV. APPLICATION

The Laplace transform technique is applicable in many fields of science and technology such as:

- Control Engineering
- Communication
- Signal Analysis and Design
- System Analysis
- Solving Differential Equations

A. *Control Systems:* Laplace transforms are used extensively in control engineering to analyze and design control systems. The transfer function of a system, which relates to the input and output of a system, is often represented in the Laplace domain. This allows engineers to study system stability, frequency response, and design controllers to achieve desired performance.

B. *Circuit Analysis:* In electrical engineering, Laplace transforms are employed to analyze and solve linear electrical circuits. By converting differential equations governing the circuit behavior into algebraic equations in the Laplace domain, circuit analysis becomes more straightforward.

C. *Signal Processing:* Laplace transforms play a crucial role in signal processing. Signals, such as audio or image data, can be transformed into the Laplace domain to analyze their frequency content and filter out unwanted components.

D. *Mechanical Systems:* Engineers use Laplace transforms to study mechanical systems like springs, masses, and dampers, simplifying their analysis and understanding their dynamic behavior.

E. *Partial Differential Equations (PDEs):* Laplace transforms are utilized to solve linear partial differential equations. By transforming a PDE into an ordinary differential equation (ODE) in the Laplace domain, the solution can be obtained more easily.

F. *Communications:* In telecommunications, Laplace transforms are used to analyze and design communication systems, including modulation, coding, and channel models.

G. *Image and Video Processing:* Laplace transforms are applied in image and video compression algorithms to transform the data into a different domain, where it can be represented more efficiently.

Laplace Transforms are put to an incredible amount of use in solving differential equations and in circuit analysis which involves components like resistors, inductors, and capacitors. Most often, during circuit analysis, the time domain equations are first written and then the Laplace Transform of the time domain equation is taken to convert it to its frequency domain equivalent. However, it is also possible to convert the circuit impedance into its frequency domain equivalent and then proceed, both of which produce the same result.

CONCLUSION

This paper thus consisted of a brief overview of what Laplace Transform is, and what is it used for. The primary use of the Laplace Transform of converting a time domain function into its frequency domain equivalent was also discussed. The Laplace transform is a versatile tool that makes it easier to analyze complex systems, by transforming complex time-domain problems into less complicated algebraic equations in the frequency domain. Its applications have significantly influenced many fields of science and engineering. The Laplace transform has made a significant contribution to technological development in many areas, including control systems, signal processing, communication, and many others by simplifying the analysis and design of linear time-invariant systems. Its many uses make it a crucial tool for engineers, scientists, and researchers who want to comprehend and work with dynamic systems.

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