INVESTIGATION OF BLOOD FLOW THROUGH STENOSED VESSEL USING NON-NEWTONIAN MICROPOLAR FLUID MODEL

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Abstract:

In this paper we investigated blood flow study in narrow vessel using non-Newtonian micro polar fluid. A mathematical model is presented here. The effects of clement stenosis on arterial blood flow features with depiction of blood by doublets stress fluid. The governing equations of flow of the present type are solved. This is closed form expression for the blood flow features. The dimensionless resistance to flow of fluid, wall shearing stresses at greatest diameter of stenosis is presented here. In this it is obtained that the wall shearing stress and resistive drag (impedance) leads with lead of tube radius for constant value of stenosis diameter. It is helps to study in medical field and physiological fluid dynamic field.

Keywords: Stress of fluids, Wall Shear Stress, Blood Flow. Mild Stenosis, Non-Newtonian fluid

1. Introduction

Blood flow system in mammalian has been the subject of scientific research for about two centuries. This types study is found in nature and life sciences. The blood flow system is the complex structure. The circulatory system and their constituent materials in life science is very important. The experimental knowledge and theoretical treatments of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human and animal physiology and for other clinical purposes and practical applications. In the recent year many researchers has been studied the blood flow characteristic through artery in the presence of stenosis. The method of the calculation of velocity rate of flow and viscous drag in the arteries when the pressure gradient is know has been studied by Womersley in (1955). Cokelet et. al. (1963) has been presented the Rheology of Human blood measurement near and at zero shear rates. Young (1968) has been studied effect of a time dependent stenosis on flow through a tube. Hemodynamic in: Annual review of fluid dynamic has been studied by Goldsmith and Skalak in (1975). Deshpande et. al. (1976) has been presented the study of laminar flow through modeled vascular stenosis. Black et. al. (1977) has been analyses the pulsatile viscous blood flow through diseased coronary arteries of Man.

A series study of this problem has been studied by Young (1979), who presented a mathematical model to analyses theoretically the effect of stenosis on flow characteristic of blood. Pulsatile flow in a heated porous channel has been studied by Bestman in (1982). Caro et. al. (1985) has been studied on blood flow near the arterial wall and arterial disease. Smith et. al. (2002) has presented an anatomically based model for transient coronary blood flow in the heart. Srivastava (2003) has been studied flow of a couple stress fluids representing blood through stenotic vessels with a peripheral layer. Mishra (2003) has been studied a mathematical model for the analysis of blood flow in arterial stenosis. In another paper Mishra and Panda (2005) has been presented the Newtonian model for blood flow through an arterial stenosis. Srivastava (2007) has been studied a theoretical model for blood flow in small vessels. Malathy & Srinivas (2008) has been studied the pulsating flow of a hydromantic fluid between permeable beds. Rathod and Tanveer (2009) have been studied the pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and Magnetics field.

It has seen that a small attention has been given to the behaviors of non-Newtonian in blood flow study. A micro polar fluid mode of blood flow through a tapered artery with a stenosis has been presented by Abdullah and Amin in (2010). Singh et. al. (2011) has been studied the analysis of blood flow behavior in Narrow Tubes through Non-linear mathematical Models. Chaubey et. al. (2012) has been studied flow of closed fitting elastics particle in very narrow vessels. Akbar (2014) has been studied Eyring prandtl fluid flow with convective boundary conditions in small intestines. Lee (2014) has been studied an adaptively refined least-squares finite element method for generalized Newtonian fluid flows using the Carreau model. Kutev et. al. (2015) has been studied approximation of the oscillatory blood flow using the Carreau viscosity model, Srinivas et. al. (2016) has been presented a note on thermal-diffusion and chemical reaction effects on MHD pulsating flow in a porous channel with slip and boundary conditions. Bary et. al. (2018) has been presented an analysis of multi-pion Hanbury Brown-Twiss correlations for the pion-emitting sources with Bose-Einstein condensation.

A condition known as stenosis contributes to an individual's risk for this type of stroke. Stenosis, in general, refers to any condition in which a blood vessel such as an artery or other tubular organ becomes abnormally narrow. For example stenosis in the arteries providing blood to brain can escort about strokes, like wise is coronary arteries, which causes coronary arteries to narrow, limiting blood flow to the heart. The blood flow problem through a stenosed artery is an important field in physiological and clinical and medical area. A number of diseases of blood vessels are being making up by general terms called stenosis. Qasim et. al. (2019) has been studied numerical simulation of MHD peristaltic flow with variable electrical conductivity and Joule dissipation using generalized differential quadrature method. Usman and Kausar (2020) has been presented Numerical solution of the partial differential equations that model the steady three-dimensional flow and heat transfer of Carreau fluid between two stretchable rotatory disks. Bary et. al. (2021) has been studied an analyses of multi-pion Bose-Einstein correlations for granular sources with coherent pion-emission droplets.

In this contemporary study, we have used a mathematical model for the blood flow through a narrow vessel introduced by assuming blood flow as a doublets stress fluid in a circular tube. The encouragement for reading these types of problem is to know the blood flow in an artery under pathological positions. The fatty plate of cholesterol and artery blocking of blood clots are firmed in the lumen of the coronary artery. The main aim of this work is to find out the analytical expressions for axial velocity, flow rate, fluid acceleration and shear stress. The present mathematical model obtains a simple form of velocity expression for blood flow. This will help not only people working in the field of physiological fluid dynamics but it help to the medical practitioners also.

2. Mathematical Formulation of the model

Let us assume that the blood flow in narrow circular tube of radius ($R = R(z)$), where z is measures along axis of tube. The pressure gradient dp/dz, represent the in the directions of x-axis of cylindrical shape. The equations of fluid motion of uniformly homogenous and susceptible fluid with couple stresses has been given by stokes as

$$\tau_{ji,j} + \rho\, F_i = \rho\, \frac{D\,V_i}{D\,t} \tag{1}$$

$$E_{ijk}\, T_{jk}^{A} + M_{ji,j} + \rho C_i = \rho K^2 \frac{D\,W_i}{D\,t} \tag{2}$$

and the corresponding essential relations are given by

$$\tau_{ij} = -P\delta_{ij} + 2\eta d_{ij} \tag{3}$$

$$\mu_{ij} = 4\eta' W_{j,i} + 4\eta'' W_{i,j} \tag{4}$$

where

η is represent the viscosity of fluids, $\eta'\, and\, \eta''$ is represent the material constants characterizing the couple stress property of the fluid, ρ represents the density of fluids, P represents the pressure, V_i is the velocity of vector, W_i represents the vorticity vector. F_i represents the body force vector per unit mass, $\tau_{ij} and\, \tau_{ij}^{A}$ represents the symmetric and anti-symmetric parts of the stress tensor, τ_{ij}, μ_{ij} represents the deviatoric part of the couple stress tensor, M_{ij}, d_{ij} is symmetric part of the velocity gradient.

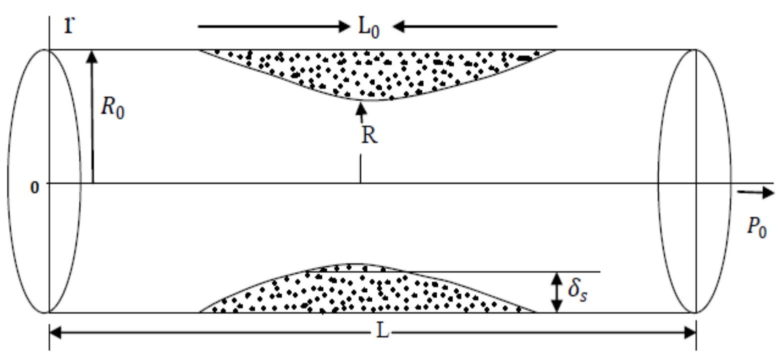

Fig. 1. Schematic diagram of Arterial Stenosis

For the pulsatile blood flow of in artery, then the equation of motion is given by equation (1) and (2), whereas in absence of body forces and the body couples stresses reduces to

$$\eta\, \nabla^2\, \mathrm{V} - \nabla \mathrm{P} + \eta' \nabla^2 (\nabla \times \nabla \times \mathrm{V}) = 0 \tag{5}$$

where

$$\mathrm{V} = \big(0, 0, \mathrm{V}(\mathrm{r}, \mathrm{z})\big),\ \ \mathrm{P} = \mathrm{P}(\mathrm{z}) \tag{6}$$

for very mild stenosis in blood flow.

The difference of velocity along z-axis has been shown in Fig (1). It can be considered as negligible as differentiate in the radial direction. It is consider that

$$\frac{\partial V}{\partial z} <<< \frac{\partial V}{\partial r} \ , \tag{7}$$

This equation can be simply corroborate, if we achieve an order of magnitude analysis, by considering the length of the artery is too large as compared to its radius. From help of equation (5), we have to obtained

$$\eta \nabla^2 V - \nabla P + \eta' \nabla^2 [(\nabla . V)\nabla - (\nabla . \nabla)V] = 0, \tag{8}$$

$$\eta \nabla^2 V - \eta' \nabla^2 [\nabla^2 V] = \Delta P, \tag{9}$$

$$\nabla^2 [\eta V - \eta' \nabla^2 V] = \frac{\partial P}{\partial z}, \tag{10}$$

$$\nabla^2 [\eta V - \eta' \nabla^2 V] = -a \tag{11}$$

Where

$$a = -\frac{\partial P}{\partial z} \quad and \quad \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) \tag{12}$$

Now integrating the equation (11), we get the following results

$$\frac{1}{r}\frac{\partial}{\partial r}[\eta V - \eta' \nabla^2 V] = -a\frac{r^2}{2} + b \tag{13}$$

Using the boundary conditions $\eta V - \eta' \nabla^2 V$ is finite, at r = 0, then we get b = 0

Then

$$V = A_2 J_0\left(\frac{r}{\beta}\right) - \frac{1}{4}\beta^2 r^2 - \frac{a}{\eta}\beta^2 A_1 - 4\frac{a\beta}{\eta}, \tag{14}$$

Where J_0 is modified Bessel function, $\beta = \sqrt{\eta'/\eta}$

Now we want to find out the constants A_1 *and* A_2, using the boundary conditions, there is no slip boundary condition for velocity at the wall i.e., V = 0 at r = R and the second boundary condition, there is no couple stress at the wall, i.e.,

$$\left\{\frac{\partial^2 V}{\partial r^2} - \frac{\overline{\eta}}{R}\frac{\partial V}{\partial r}\right\}_{r=R} = 0, \tag{15}$$

Where $\bar{\eta} = \frac{\eta'}{\eta}$,

Here, it is justified because for the practical observations, which show that the suspended particle has the propensity to move aside from boundary, leaving a clear fluid nears the wall. The couple stress arises only in suspension. Here the clear fluid cannot be longer support the couples stress at the boundary layer. The condition (15) has been much used by many researchers in study of blood flow behavior wherever the couple stress effects are included has been studied. After calculating the constant $A_1\ and\ A_2$, the velocity profile of blood flow is completely calculated as

$$V = \frac{R^2 a}{4\eta}\left[1 - \frac{r^2}{R^2} - H\left\{1 - \frac{J_0\left(\frac{r}{\beta}\right)}{J_0\left(\frac{R}{\beta}\right)}\right\}\right], \tag{16}$$

where

$$H = \frac{2(1-\overline{\eta})}{\left(\frac{R}{\beta}\right)^2}\frac{1}{\left\{1-\beta\left(\frac{1+\overline{\eta}}{R}\right)\frac{J_1\left(\frac{R}{\beta}\right)}{J_0\left(\frac{R}{\beta}\right)}\right\}} \tag{17}$$

Now equation (16) reduces

$$V = V^*\left[1 - \frac{r^2}{R^2} - H\left\{1 - \frac{J_0\left(\frac{r}{\beta}\right)}{J_0\left(\frac{R}{\beta}\right)}\right\}\right], \tag{18}$$

Where

$$V^* = \frac{R^2 a}{4\eta}\ , \tag{19}$$

This type of geometry of stenosis which geometry is given in figure 1 has been proposed by Young in 1968.

$$\frac{R(z)}{R_0} = \frac{1-\delta_s}{2R_0}\left[1 + \frac{\cos 2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)\right], \tag{20}$$

$$d \leq z \leq L_0 + d = 1, otherwise \tag{21}$$

Here

R_0 is constant radius, L_0 represents length of stenosis, δ_s represents maximum height of the stenosis, $R(z)$ represents the radius of the tube with stenosis, It is considered that $\delta_s << R_0$, i.e., (very mild stenosis). Let the volumetric rate of blood flow is represented by T, which is given by

$$T = 2\pi \int_0^R V\, r\, dr, \tag{22}$$

$$T = \frac{\pi R^4 (\frac{dP}{dz})}{8\eta}\left[1 - \frac{2H}{R/\beta}\left\{\frac{R}{\beta} - \frac{2J_1(R/\beta)}{J_0(R/\beta)}\right\}\right], \tag{23}$$

$$\frac{dP}{dz} = -\frac{8\eta T}{\pi R^4}\left[1 - \frac{2H}{R/\beta}\left\{\frac{R}{\beta} - \frac{2J_1(R/\beta)}{J_0(R/\beta)}\right\}\right]^{-1} \tag{24}$$

Now the average velocity of fluid represented as

$$\bar{V} = \frac{T}{\pi R^2} = -\frac{R^2 (\frac{dP}{dz})}{8\eta}\left[1 - \frac{2H}{R/\beta}\left\{\frac{R}{\beta} - \frac{2J_1(R/\beta)}{J_0(R/\beta)}\right\}\right], \tag{25}$$

Shear stress at wall, when $r = R$ is explain by

$$\tau_R = \eta\left(\frac{\partial V}{\partial r}\right)_{r=R}, \tag{26}$$

$$\tau_R = \left[\frac{R\,(dP/dz)}{2}\left\{1 - \frac{RH}{2}\left\{\frac{J_1(R/\beta)}{J_0(R/\beta)}\right\}\right\}\right], \tag{27}$$

From equation (23) and (27), we obtained

$$\frac{\tau_R}{T} = \frac{-4\eta\left[1 - \frac{RH}{2}\left\{\frac{J_1(R/\beta)}{J_0(R/\beta)}\right\}\right]}{\pi R^3\left[1 - \frac{2H}{R/\beta}\left\{\frac{R}{\beta} - \frac{2J_1(R/\beta)}{J_0(R/\beta)}\right\}\right]}, \tag{28}$$

Let us assumed τ_N is normalized with steady state flow solution given by equation

$$\tau_N = -\frac{R_0}{2}\left(\frac{dP}{dz}\right)_0, \tag{29}$$

Then equation for wall shear stress is

$$\bar{\tau} = \frac{\tau_R}{\tau_N} = -\frac{R}{R_0}\left[1 - \frac{RH}{2}\left\{\frac{J_1(R/\beta)}{J_0(R/\beta)}\right\}\right]\frac{dP/dz}{(dP/dz)_0}, \tag{30}$$

In equation (30) substitute the value of $\delta_s = R_0 + R$ because maximum height of stenosis is sum of constant radius and it actual radius, and taking the modulus of the given data, because it is no negative values.

$$|\bar{\tau}| = \left|\frac{\tau_R}{\tau_N}\right| = \left(1 - \frac{\delta_s}{R_0}\right)\left[1 - \frac{RH}{2}\left\{\frac{J_1(R/\beta)}{J_0(R/\beta)}\right\}\right]\frac{dP/dz}{(dP/dz)_0}, \tag{31}$$

The resistance impedance to the flow is defined by

$$J = -\left(\frac{dP/dz}{T}\right), \tag{32}$$

In the above equation RHS, putting the value from above equation (23) and (24), we get

$$J = - \frac{8\eta}{\pi R^4}\left[1 - \frac{2H}{R/\beta}\left\{\frac{R}{\beta} - \frac{2I_1(R/\beta)}{I_0(R/\beta)}\right\}\right]^{-1}, \tag{33}$$

$$\frac{J}{a} = \frac{1}{T} \tag{34}$$

3. Results and Discussions

The quantity of on the spot volumetric flow amount respective to steady flow rate value of poiseulle is the oppositely to the resistive encumbrance (impedance) normalized to the steady state volume. It is found that a reduction of rate of flow comparison to the steady state flow leads to resistive impedance. The table 1 shows that the difference of resistive impedance with the tube radius for difference value of stenosis height has been presented. From figure 2, it is also seen that corresponding impedance increases with the increase of tube radius for constant height of stenosis. We can conclude it that the resistive wall stresses and impedance leads for a particular value of the stenosis height. The change of wall shears stress with tube radius R/β has been presented in this figure. The table 2 presented as wall shear stress for a constant value of stenosis height δ_s/R_0, it is found that the wall shear stress leads with lead of tube radius. This shows that that the pair stresses of blood fluid is extra delicate or sensitive to stenosis in differentiation of the Newtonian fluid. From figure 3, it is seen that walls shear stress is decreases with the increase of stenosis height δ_s/R_0. Now we can say that the effect of a mild stenosis on the whole flow feature is small. But the feature of non-Newtonian of blood flow shows that mild stenosis leads the resistance to flow significantly. Hence it will be marked cutback in the specific vascular blood provided by the stenotic arteries. It can also say that which change of abnormal cell proliferation.

TABLE.1: Table for variation of impedance with respect to tube at constant value of stenosis height. ($\frac{J}{a}$ *for constant value of stenosis Hight* δ_s/R_0)

R/β	$\delta_s/R_0 = 0.00$	$\delta_s/R_0 = 0.03$	$\delta_s/R_0 = 0.05$
0	1.02	1.09	1.66
2	5.40	6.04	6.96
4	11.01	12.41	13.86
6	16.46	18.06	20.05
8	19.86	22.03	23.82
10	24.08	26.56	28.46

TABLE.2: Table for variation of wall shear stresses at constant stenosis Height ($|\bar{\tau}|$ *for constant value of stenosis height* δ_s/R_0)

R/β	$\delta_s/R_0 = 0.00$	$\delta_s/R_0 = 0.05$	$\delta_s/R_0 = 0.09$
0	0.62	0.58	0.502
2	0.96	0.77	0.67
4	6.04	5.68	4.98
6	13.86	12.98	12.76
8	27.08	26.04	24.98
10	52.12	21.08	47.16

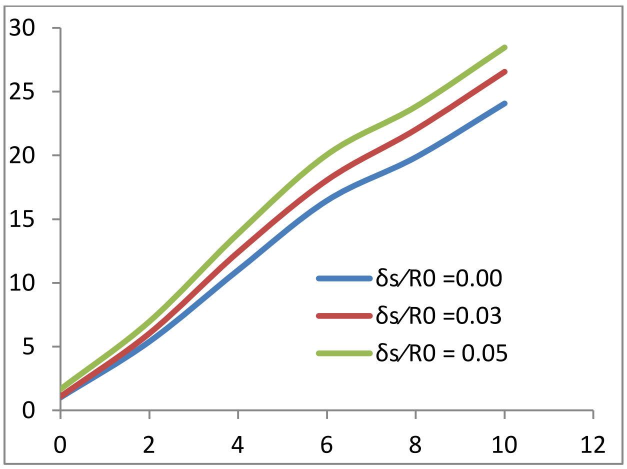

Fig.2. Graph between R/β & $\frac{J}{a}$, where R/β represented as independent variable in x- axis and $\frac{J}{a}$ (Normalized impedance) as dependent variable in y-axis, and variation of stenosis height is presented in this graph.

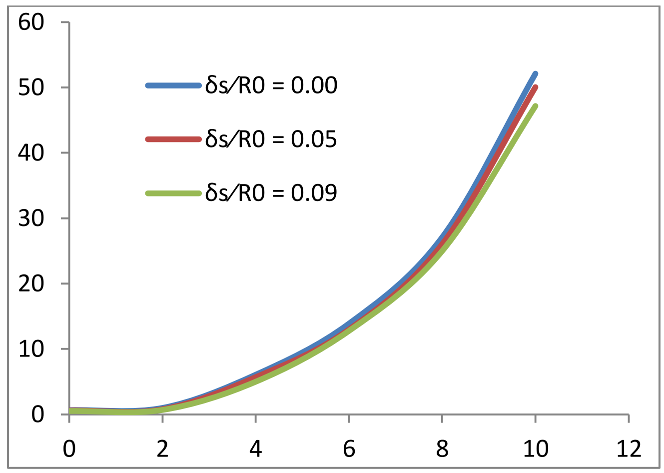

Fig.3. Graph between R/β & $|\bar{\tau}|$, where R/β represented as independent variable in x- axis and $|\bar{\tau}|$ (wall shearing stress) as dependent variable in y-axis, and variation of stenosis height is presented in this graph.

4. Conclusions

The main objective is to study the investigation of blood flow through stenosed vessel using non-Newtonian micro polar fluid. The effect of stenosis on wall shear stress and resistive impedance has been studied, it is seen that increase of stenosis heights, shearing stress and resistive impedance has been decreases, whereas increase of tube radius it is increases. The application of this model is in the clinical, medical and physiological field, for the precaution of clotting of blood in vessels.

5. References

[1]. Abdullah I. and N. Amin (2010); 'A micro polar fluid model of blood flow through a tapered artery with a stenosis,' Mathematical Methods in the Applied Sciences, vol. 33, no. 16, pp. 1910–1923.

[2]. Akbar N.S. (2014); 'Eyring Prandtl fluid flow with convective boundary conditions in small intestines.' International Journal of Biomathematics, Vol. 6, no. 5, pp. 1–13.

[3]. Bary G., Ru P. and W. Zhang (2018); 'Analyses of multi-pion Hanbury Brown-Twiss correlations for the pion-emitting sources with Bose-Einstein condensation.' Journal of Physics G: Nuclear and Particle Physics, Vol. 45, Article ID 065102.

[4]. Bary G., Zhang W., Ru P. and J. Yang (2021); 'Analyses of multi-pion Bose-Einstein correlations for granular sources with coherent pion-emission droplets.' Chinese Physics C, Vol. 21, Article ID 024106.

[5]. Bestman, A. R. (1982); 'Pulsatile flow in a heated porous channel.' Int. J. Heat Mass Trans. Vol. **25**(5), 675–682.

[6]. Black L. D., Radbill J.R. and D. W. Crawford (1977); 'Analysis of pulsatile viscous blood flow through diseased coronary arteries of Man.' Journal of Biomechanics, Vol. 10, pp. 339.

[7]. Caro C.G., Parker K.H., Fish P.J and M. J. Lever (1985); 'Blood flow near the arterial wall and arterial disease.' Clin. Hemorheol, Vol. 5, pp. 849 – 871.

[8]. Chaubey A.K., Srivastava A. and R. Yadav (2012); 'Flow of closely fitting elastics particle in very narrow vessels.' Jour PAS, Vol. 18, (Mathematical Science), pp. 1 – 9.

[9]. Cokelet G. R., Merrill E. W., Gilliland E. R and H. Singh (1963); 'The Rheology of Human Blood measurement near and at zero Shear Rate.' Trans. Soc, Rheol, Vol. 7, pp. 303 – 317.

[10]. Deshpande M.D., Giddens D. P. and R.F. Mason (199=76); 'Study of laminar flow through modeled vascular stenosis.' Journal of Biomech, Vol. 9, pp 165.

[11]. Goldsmith H. L. and R. Skalak (1975); 'Hemodynamic In: Annual Review of Fluid Dynamics.' Annual Review Inc. Pato, Alto. Pub. Pp. 231 – 247.

[12]. Kutev N., Tabakova S. and S. Radev (2015); 'Approximation of the oscillatory blood flow using the Carreau viscosity model,' in Proceedings of the International Conference on Mechanics—Seventh Polyakhov's Reading, pp. 1–4.

[13]. Lee H.C. (2014); 'An adaptively refined least-squares finite element method for generalized Newtonian fluid flows using the Carreau model.' SIAM Journal on Scientific Computing, Vol. 36, no. 1, pp. 193–218.

[14]. Malathy, T. & Srinivas, S.(2008); 'Pulsating flow of a hydromantic fluid between permeable beds.' Int. Commun. *Heat Mass Transfer*. Vol. **35**(5), pp. 681–688.

[15]. Mishra B.K. (2003); ‘A mathematical model for the analysis of blood flow in arterial stenosis.’ The Mathematical Education, Vol. 37,(4), pp. 176 – 181.

[16]. Mishra B.K. and T.C. Panda (2005); ‘Newtonian model for blood flow through an arterial stenosis.’ The Mathematics Education, Vol. 39(3), pp. 151 -160.

[17]. Smith N. P., Pullan A. J. and P.J. Hunter (2002); ‘An anatomically based model for transient coronary blood flow in the heart.’ Siam Jour. Appl. Math., Vol. 62, No. 3, pp. 990 – 1018.

[18]. Qasim M., Ali Z., Wakif A. and Z. Boulahia (2019); ‘Numerical simulation of MHD peristaltic flow with variable electrical conductivity and Joule dissipation using generalized differential quadrature method.’ Communications in Theoretical Physics, Vol. 71, no. 509, pp. 509–518.

[19]. Rathod V.P. and S. Tanveer (2009); ‘Pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and Magnetics field.’ Bull Malays. Math. Sci. Sco.(2), Vol. 32(2), pp. 245 - 259

[20]. Singh N.L., Kumar M., Vidita, Vandana and R. Yadav (2011); ‘Analysis of blood flow behavior in Narrow Tubes through Non-linear mathematical Models.’ Jour PAS, Vol. 17, (Mathematical Science), pp. 332 – 345.

[21]. Srivastava V. P. (2007); ‘A theoretical model for blood flow in small vessels.’ Applications and Applied Mathematics Vol. 2, No. 1, pp. 51 – 65.

[22]. Srivastava V.P. (2003); ‘Flow of a couple stress fluid representing blood through stenotic vessels with a peripheral layer.’ Indian Journal of Pure and Applied Maths, Vol. 34 (12), pp. 1727 – 1740.

[23]. Srinivas, S., Malathy, T. & Reddy, A. S. (2016); ‘A note on thermal-diffusion and chemical reaction effects on MHD pulsating flow in a porous channel with slip and boundary conditions.’ Jour. King Saud. Univ. Vol. **28**(2), 213–281.

[24]. Usman A. G. and S. Kausar (2020); ‘Numerical solution of the partial differential equations that model the steady three-dimensional flow and heat transfer of Carreau fluid between two stretchable rotatory disks,’ Numerical Methods for Partial Differential Equations, pp. 1–29.

[25]. Womersley J.R. (1955); ‘Method for the calculation of velocity rate of flow and viscous drag in the arteries when the pressure gradient is known.’ Jour of Physiology, Vol. 127, pp. 553 – 563.

[26]. Young D.F. (1968); ‘Effect of a Time Dependent Stenosis on Flow through a Tube.’ Journal of Engang. Int. Trans. Am. Soc. Mech. Engrs., Vol. 98, pp 248.

[27]. Young D. F. (1979); ‘Fluid mechanics of arterial stenosis.’ Jour. Biomechanics Engg. Trans. ASME, Vol. 101, pp. 157 – 175.