A FUZZY MATHEMATICAL MODEL FOR THE PARATHYROID HORMONE SECRETION IN HEALTH USING LOG GAMMA DISTRIBUTION

Dr.S. Prakasam[1], Dr.A.Venkatesh[2] & Dr.P.Syamala[3]

Assistant Professors, Department of Mathematics,

A. V. V. M. Sri Pushpam College, Poondi, Thanjavur (Dt), Tamilnadu.

Abstract

In this article, the formula for mean and variance of fuzzy log gamma distribution was developed and used to find the changing aspects of PTH secretion in strength and ancillary hyperparathyroidism.

Keywords

Mathematical modeling, Fuzzy set, Fuzzy mean value, Fuzzy variance value, log gamma distribution, parathyroid hormone.

2000 Mathematics Subject Classification

00A71, 03E72, 60E05, 62-07,62J10, 62P10

1. Introduction

Mathematical modeling is an increasingly used in complex mathematical models, engineering, computational science, physics, and biology [8]. It has a long history that predates biology. Biologists use 'real-world' models that are easy to touch. Biologists also use models of philosophy [2]. G-MVLG (multivariate log-gamma distribution) is an elastic diffusion. Because of this property, the distribution is successfully used as a joint prior distribution in Bayesian analysis.

It is noteworthy that parathyroid hormone secretion is inhibited by serum calcium concentration, and during its calcemic activities, parathyroid hormone inhibits serum calcium concentration. Thus, instead of hypocalcemia, parathyroid hormone secretion restores the serum calcium concentration to normal levels. The composition of parathyroid hormone in humans is less well studied, with modern clinical studies restricted to the assessment of parathyroid hormone response to changes in serum calcium absorption. Changes in serum calcium absorption usually result from the use of low and high calcium dialysate concentrations in normal humans with calcium chelating agents, calcium compounds, and during dialysis treatment in hemodialysis patients [1]. Upregulation of parathyroid hormone by hypercalcemia results in increased renal secretion of calcium through both parathyroid hormone levels in the loop of Henle and the calcium-sensing receptor, thereby returning serum calcium to normal levels. An elevated parathyroid hormone value also reduces calcium excretion from bone, renal phosphorus excretion, and calcitriol, all of which are used to normalize serum calcium concentration [4]. On the other hand, when hypocalcemia develops, the effect of parathyroid hormone is to normalize the serum calcium value due to increased calcium excretion from bone. The latter effect is neutralized when the serum phosphorus concentration decreases [4].

Basal parathyroid hormone, a measure of parathyroid hormone inspiration, is derived from the general energy to produce parathyroid hormone. When hypocalcemia is present, the parathyroid gland indicates that it is using too little calcium. The set point of calcium is evident from parathyroid hormone and serum calcium absorption in the midpoint of the calcium curve. Minimal parathyroid hormone is the lowest parathyroid hormone absorption measured at the onset of hypercalcemia. Even in healthy individuals, hypercalcemia results in decreased parathyroid hormone secretion. Postprandial parathyroid hormone values increase [9] when fed a single diet high in phosphate, even in rats and humans. Due to disorders like CKD, vitamin D deficiency and hyperparathyroidism, the amount of parathyroid gland in our body increases from primary hyperparathyroidism. More often than not, the adenoma leads to hypocalcemia resulting in maximal parathyroid hormone changes [3], [6], [7], [10].

Baseline parathyroid hormone values ranged from 10 to 20 times normal in studies of primary hyperparathyroidism in India and China. But in the US they differ by 1.5 to 2.0 times. In these patients, the parathyroid gland is enlarged and the parathyroid hormone value increases 10- to 20-fold. In the first study when iPTH was assessed in volunteers over 60 or 120 minutes, when the same amount of hypocalcemia was induced, there was no difference in parathyroid hormone levels after hypocalcemia [11]. In the following study, a gradual decrease in parathyroid hormone followed a typical linear decrease of serum calcium over 120 min. Finally, elevated parathyroid hormone levels reach a peak when serum calcium absorption decreases rapidly at certain times. But at the onset of hypocalcemia the parathyroid hormone value remains unchanged [5]. Episodic, elevated parathyroid hormone values in serum calcium. In this paper, fuzzy models are used to calculate fuzzy expected values and fuzzy variance values of differences in parathyroid hormone values using fuzzy log-gamma distribution.

2. Fuzzy Log-Gamma Distribution

A variable X is Log-Gamma distributed if its normal log is Gamma distributed. The Probability density function of Log-Gamma distribution is given by

$$f(x;\gamma,\mu) = \frac{(In[x-\sigma+1])^{\gamma-1}(x-\sigma+1)^{-\left(\frac{1+\mu}{\mu}\right)}}{\mu^{\gamma}\Gamma(\gamma)}$$

Where the variable $x \geq \sigma$ and the parameters $\gamma, \mu > 0$ all are real numbers

The alpha cut of fuzzy expected function is $\bar{E}(x) = \{\bar{E}_l(x), \bar{E}_u(x)\}$

Where $\bar{E}_l(x) = \min\{(1-\bar{\mu})^{-\bar{\gamma}} + \sigma - 1\}$ and

$\bar{E}_u(x) = \max\{(1-\bar{\mu})^{-\bar{\gamma}} + \sigma - 1\}$.

The alpha cut of fuzzy variance function is $\bar{V}(x) = \{\bar{V}_l(x), \bar{V}_u(x)\}$

Where $\bar{V}_l(x) = \min\left\{(1-2\bar{\mu})^{-\bar{\gamma}} + (1-\bar{\mu})^{-\overline{2\gamma}}\right\}$ and

$\bar{V}_u(x) = \max\left\{(1-2\bar{\mu})^{-\bar{\gamma}} + (1-\bar{\mu})^{-\overline{2\gamma}}\right\}$

3. Application

Let us consider a study by Arnold J. Felsenfeld et al [1] for the changing aspects of PTH secretion in strength and ancillary hyperparathyroidism. Blood samples for measurements of intact PTH were drawn at 20 minute interval. The levels of intact parathyroid hormone obtained during 120 minutes were taken.

Table.3.A level of intact parathyroid hormone by a linear induction

Time (min)	0	20	40	60	80	100	120
Intact PTH(pg/ml)	20	40	50	52	58	68	60

Table.3.B level of intact parathyroid hormone by an episodic induction

Time (min)	0	20	40	60	80	100	120
Intact	20	44	42	48	52	54	58

PTH(pg/ml)							

4. Results

The parameters of Log-Gamma distribution for Table.3.A are scale parameter γ = 87.851 and shape parameter μ = 0.04378.

Let the equivalent fuzzy triangular numbers are $\bar{\gamma}$ = [87.0678, 87.851, 88.3983] and $\bar{\mu}$ = [0.02246, 0.04378, 0.05612] and the corresponding α- cut are given by $\bar{\gamma}$ = [87.0678 + 0.7832α, 88.3983- 0.5473α] and $\bar{\mu}$ = [0.02246 + 0.02132α, 0.05612 - 0.01234α].

Table 4.A Fuzzy Expected value for lower and upper alpha values

α	lower γ	lower μ	upper γ	upper μ	σ	lower E(X)	upper E(X)
0	87.0678	0.02246	88.3983	0.05612	10	16.2271	173.9353
0.1	87.14612	0.024592	88.34357	0.054886	10	17.75731	155.4919
0.2	87.22444	0.026724	88.28884	0.053652	10	19.61957	139.1491
0.3	87.30276	0.028856	88.23411	0.052418	10	21.88766	124.6633
0.4	87.38108	0.030988	88.17938	0.051184	10	24.65212	111.8199
0.5	87.4594	0.03312	88.12465	0.04995	10	28.02417	100.4293
0.6	87.53772	0.035252	88.06992	0.048716	10	32.1405	90.32417
0.7	87.61604	0.037384	88.01519	0.047482	10	37.16929	81.35692
0.8	87.69436	0.039516	87.96046	0.046248	10	43.31756	73.39709
0.9	87.77268	0.041648	87.90573	0.045014	10	50.84037	66.32946
1	87.851	0.04378	87.851	0.04378	10	60.05223	60.05223

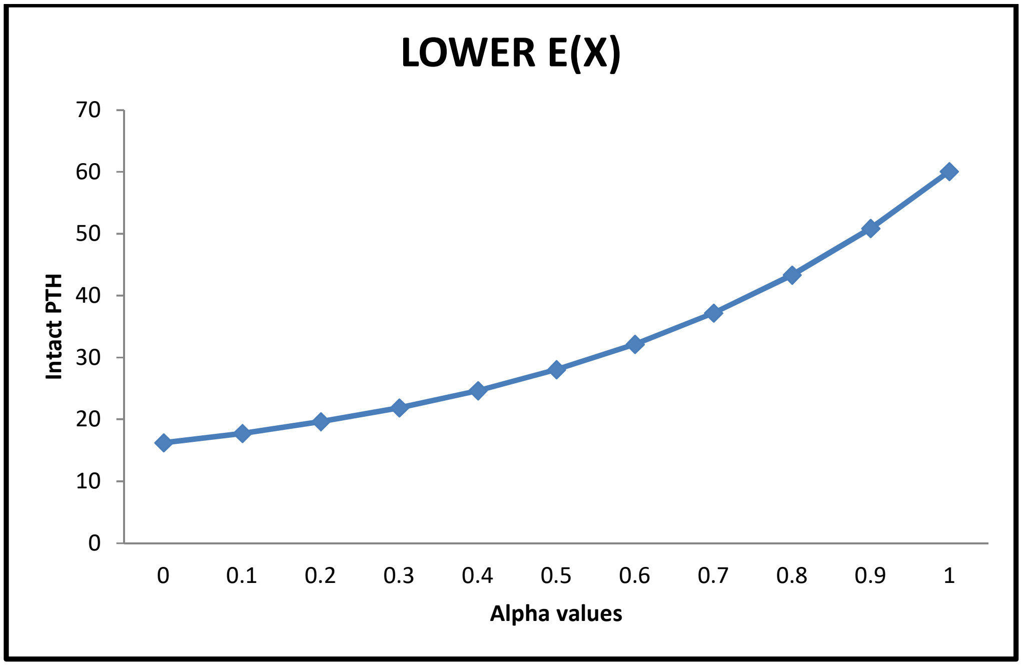

Fig.4.A Fuzzy expected value for lower alpha values

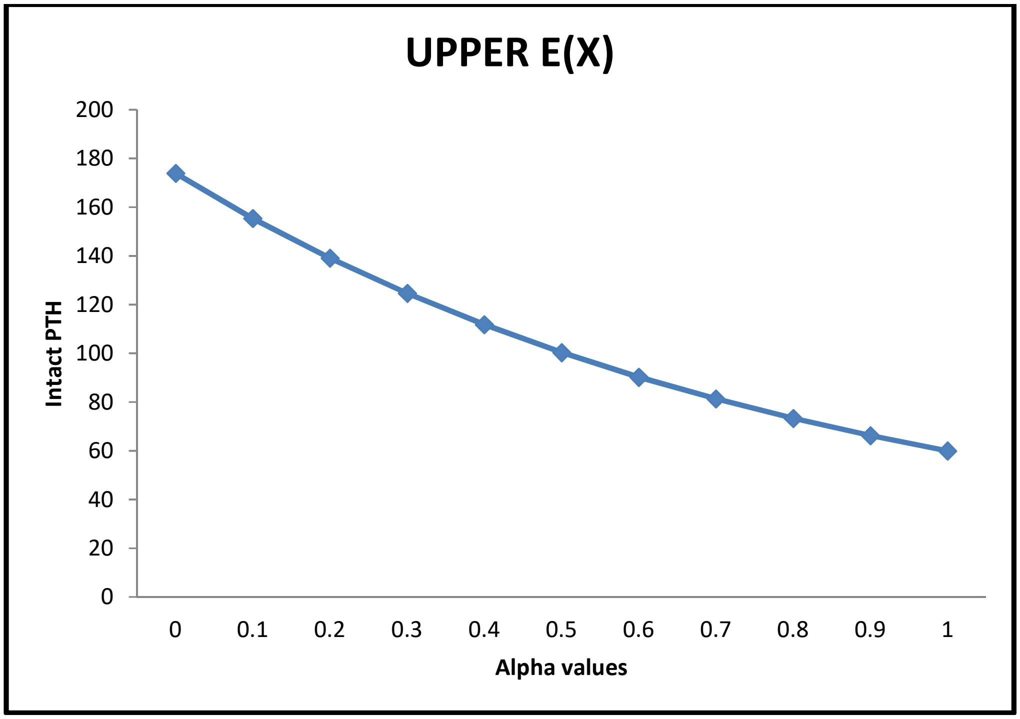

Fig.4.B Fuzzy expected value for upper alpha values

Table 4.B Fuzzy Variance value for lower and upper alpha values

α	lower γ	lower μ	upper γ	upper μ	σ	lower E(X)	upper E(X)
0	87.0678	0.02246	88.3983	0.05612	10	54.70752	64407.13
0.1	87.14612	0.024592	88.34357	0.054886	10	81.07305	50382.65
0.2	87.22444	0.026724	88.28884	0.053652	10	120.4527	39446.05
0.3	87.30276	0.028856	88.23411	0.052418	10	179.4123	30909.97
0.4	87.38108	0.030988	88.17938	0.051184	10	267.9055	24241.68
0.5	87.4594	0.03312	88.12465	0.04995	10	401.0595	19027.99
0.6	87.53772	0.035252	88.06992	0.048716	10	601.9225	14948.11
0.7	87.61604	0.037384	88.01519	0.047482	10	905.7023	11752.75
0.8	87.69436	0.039516	87.96046	0.046248	10	1366.32	9248.053
0.9	87.77268	0.041648	87.90573	0.045014	10	2066.574	7283.086
1	87.851	0.04378	87.851	0.04378	10	3133.94	5740.27

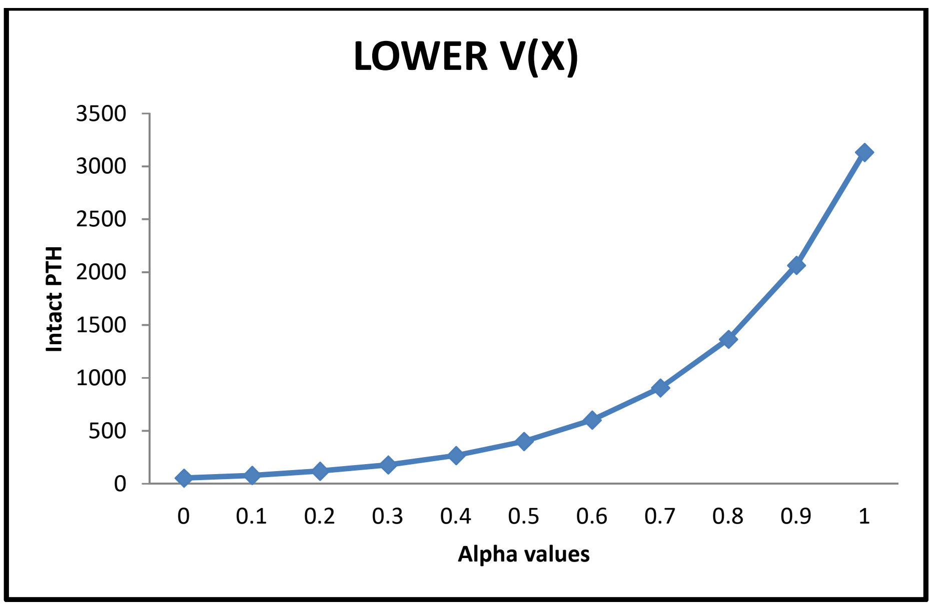

Fig.4.C Fuzzy Variance value for lower alpha values

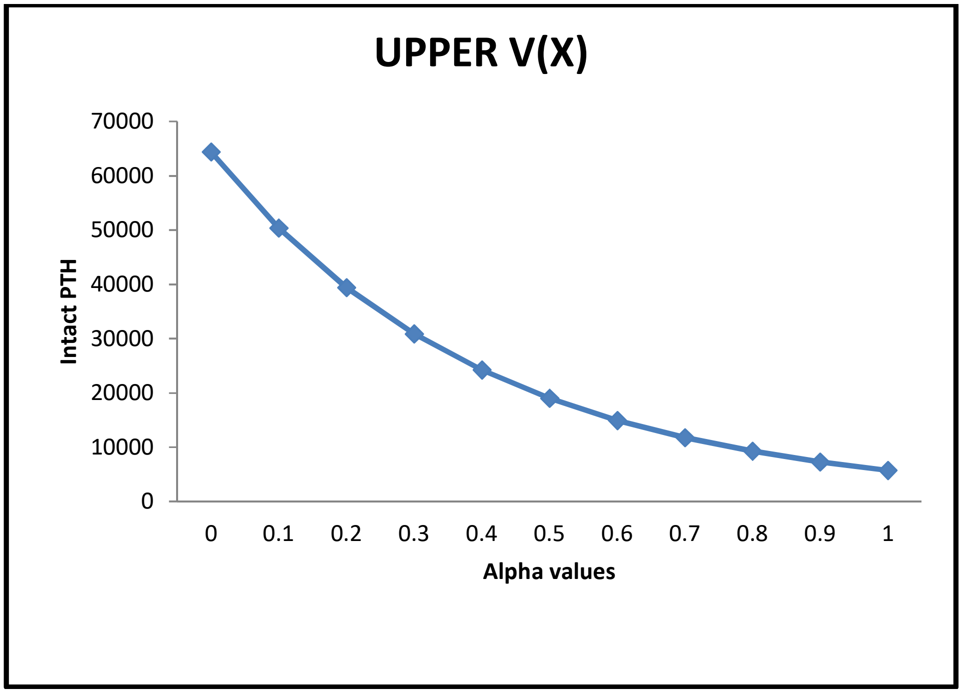

Fig.4.D Fuzzy Variance value for upper alpha values

Also the parameters of Log-Gamma distribution for Table.3.B are scale parameter γ = 109.87 and shape parameter μ = 0.03431.

Let the corresponding fuzzy triangular numbers are $\bar{\gamma}$ = [109.0855, 109.87, 110.3744] and $\bar{\mu}$ = [0.01106, 0.03431, 0.04678] and the corresponding α- cut are given by $\bar{\gamma}$ = [109.0855 + 0.7845α, 110.3744 – 0.5234α] and $\bar{\mu}$ = [0.01106 + 0.02325α, 0.04678 – 0.01247α].

Table 4.C Fuzzy expected value for lower and upper alpha values

α	**lower γ**	**lower μ**	**upper γ**	**upper μ**	**σ**	**lower E(X)**	**upper E(X)**
0	109.0855	0.01106	110.3744	0.04678	10	12.36426	206.9448
0.1	109.164	0.013385	110.3221	0.045533	10	13.3537	179.9296
0.2	109.2424	0.01571	110.2697	0.044286	10	14.63965	156.6494
0.3	109.3209	0.018035	110.2174	0.043039	10	16.31259	136.5813
0.4	109.3993	0.02036	110.165	0.041792	10	18.49112	119.2764
0.5	109.4778	0.022685	110.1127	0.040545	10	21.33086	104.3496
0.6	109.5562	0.02501	110.0604	0.039298	10	25.03617	91.46972
0.7	109.6347	0.027335	110.008	0.038051	10	29.87568	80.35262
0.8	109.7131	0.02966	109.9557	0.036804	10	36.20293	70.75391
0.9	109.7916	0.031985	109.9033	0.035557	10	44.48359	62.46357
1	109.87	0.03431	109.851	0.03431	10	55.33169	55.30097

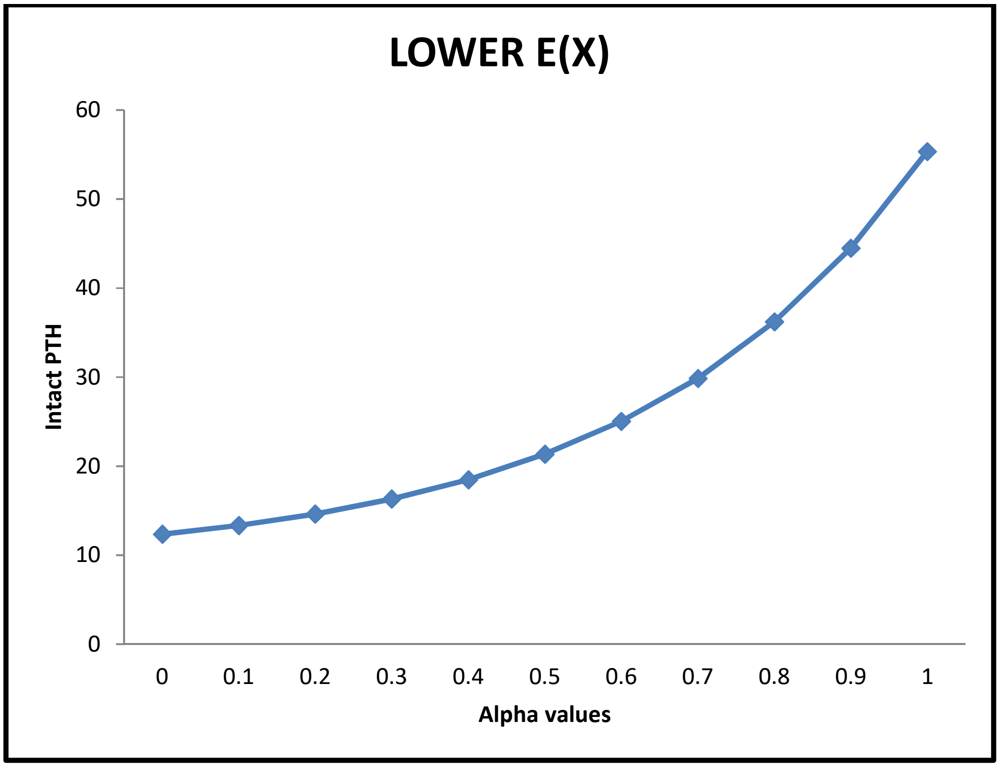

Fig.4.E Fuzzy expected value for lower alpha values

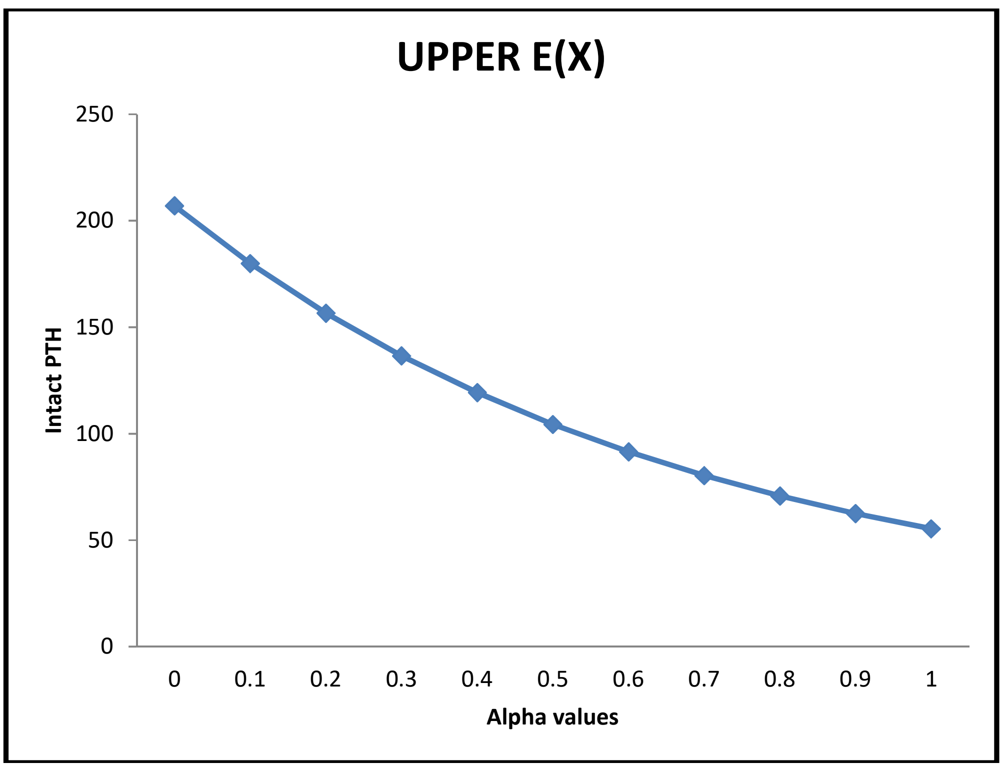

Fig.4.F Fuzzy expected value for upper alpha values

Table 4.D Fuzzy Variance value for lower and upper alpha values

α	lower γ	lower μ	upper γ	upper μ	σ	lower E(X)	upper E(X)
0	109.0855	0.01106	110.3744	0.04678	10	11.56207	90312.09
0.1	109.164	0.013385	110.3221	0.045533	10	19.3922	66783.07
0.2	109.2424	0.01571	110.2697	0.044286	10	32.73474	49431.78
0.3	109.3209	0.018035	110.2174	0.043039	10	55.50169	36623.64
0.4	109.3993	0.02036	110.165	0.041792	10	94.45235	27159.93
0.5	109.4778	0.022685	110.1127	0.040545	10	161.2976	20160.62
0.6	109.5562	0.02501	110.0604	0.039298	10	276.3917	14979.04
0.7	109.6347	0.027335	110.008	0.038051	10	475.2291	11139.49
0.8	109.7131	0.02966	109.9557	0.036804	10	819.9192	8291.718
0.9	109.7916	0.031985	109.9033	0.035557	10	1419.519	6177.584
1	109.87	0.03431	109.851	0.03431	10	2466.191	4606.641

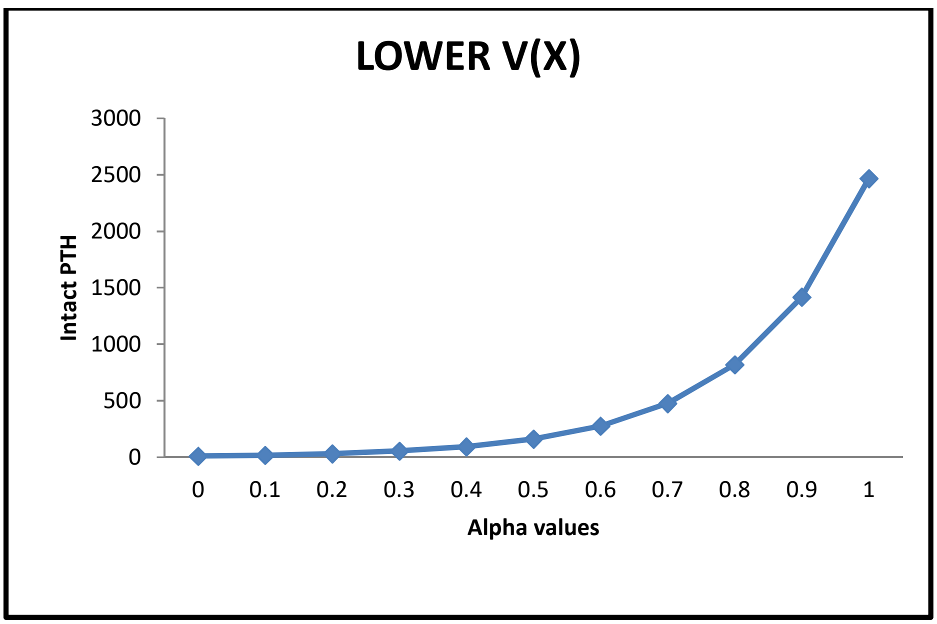

Fig.4.G Fuzzy Variance value for lower alpha values

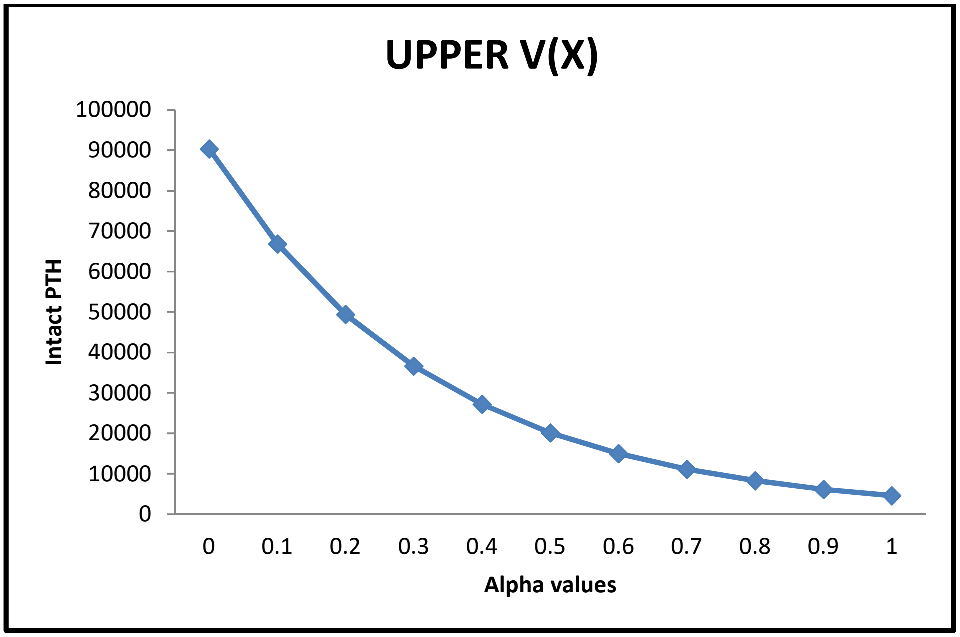

Fig.4.H Fuzzy Variance value for upper alpha values

5. Conclusion

We exhibit the assessment of dynamics of parathyroid hormones secretion in health and secondary hyperparathyroidism by the guesstimate of expectation and variance with binary parameter Log- Gamma distribution. The result shows that the fuzzy expected increases in lower alpha cuts and decreases in upper alpha cuts and also the fuzzy variance are increases in lower alpha cuts and decreases in upper alpha cuts for the effect of the linear and episodic induction in serum calcium produced higher parathyroid hormone values.

References

[1] Arnold J.Felsenfeld, Mariano Rodriguez, Escolastico Aguilera-Tejero, “Dynamics of Parathyroid Hormone Secretion in Health and secondary hyperparathyroidism’’, Clin J Am Soc Nephrol 2 : 1283-1305,2007.

[2] Brain Ingalls, “Mathematical modeling in systems biology: An Introduction’’, University of waterloo, 2012.

[3] Cloutier M, Gascon-Barre M, D’Amour P, “Chronic adaptation of dog parathyroid function to a lowcalcium -high sodium-Vitamin D-deficient diet’’ J Bone Miner Res 7: 1021– 1028, 1992.

[4] Felsenfeld AJ, Levine BS, “Milk alkali syndrome and the dynamics of calcium homeostasis’’, Clin J Am Soc Nephrol 1: 641–654, 2006.

[5] Grant FD, Conlin PR, Brown EM, “Rate and concentration dependence of parathyroid hormone dynamics during stepwise changes in serum ionized calcium in normal humans’’, J Clin Endocrinol Metab 71: 370–378, 1990.

[6] Malberti F, Farina M, Imbasciati E, “The PTH-calcium curve and the set point of calcium in primary and secondary hyperparathyroidism’’, Nephrol Dial Transplant 14: 2398– 2406, 1999.

[7] Messa P, Vallone C, Mioni G, Geatti O, Turrin D, Passoni N, Cruciatti A, “Direct in vivo assessment of parathyroid hormone-calcium relationship curve in renal patients’’, Kidney Int 46: 1713–1720, 1994.

[8] Murray J.D, “Mathematical Biology’’, springer, Vol-12 (II), 2002.

[9] Nishida Y, Taketani Y, Yamanaka-Okumura H, Imamura F, Taniguchi F, Sato T, Shuto E, Nashiki K, Arai H, Yamamoto H, Takeda E, "Acute effect of oral phosphate loading on serum fibroblast growth factor 23 levels in healthy men'', Kidney Int 70: 2141–2147, 2006.

[10] Ramirez JA, Goodman WG, Gornbein J, Menezes C, Moulton L, Segre GV, Salusky IB, "Direct in vivo comparison of calcium-regulated parathyroid hormone secretion in normal volunteers and patients with secondary hyperparathyroidism'', J Clin Endocrinol Metab 76: 1489–1494, 1993.

[11] Rao DS, Agarwal G, Talpos GB, Phillips ER, Bandeira F, Mishra SK, Mithal, "Role of vitamin D and calcium nutrition in disease expression and parathyroid tumor growth in primary hyperparathyroidism: A global perspective'', J Bone Miner Res 17[Suppl 2]: N75–N80, 2002.

[12] Rao DS, Honasoge M, Divine GW, Phillips ER, Lee MW, Ansari MR, Talpos GB, Parfitt AM, "Effect of vitamin D nutrition on parathyroid adenoma weight: Pathogenetic and clinical implications'', J Clin Endocrinol Metab 85: 1054– 1058, 2000.