The Graphical domination theory has certain limitations with Coding

P.R.Rajesh Kumar[1], P.Raveendra Nath[2]

1..Lecturer in Computer science & Engineering Sri Krishnadevaraya University College of Engineering and Technology, S.K.University, Anantapur - 515 003, A.P., India

2..Lecturer in Mathematics, Sri Krishnadevaraya University College of Engineering and Technology, S.K.University, Anantapur - 515 003, A.P., India

Abstract: I want to talk about a few limitations in the theory of dominances in this session. Along with highlighting their existence through a number of examples, I would also like to highlight the goal of this study, which is to generalize the traditional representation of dominance in graphs to incorporate a specified level of redundancy in domination. When a graph's set D of vertices has at least n-embers, we refer to it as a onn-dominating set of each vertex not in Disadjacent. The cardinality at a minimum N-dominating set G is defined as the then-domination number Yn(G). We also examine the behavior of Yn(G) to derive both bounds and exact values for the parameter.

Key Words: Graph,Dominating,Chromatic number

1. Introduction

If every vertex in a graph G is adjacent to a member of a non-empty subset D of the vertex set V (G), then D is a dominant set. if both u and v belong to V (G). We state that "v" is dominated by "u," and "u" dominates "v."A minimum dominating set is a dominating set that has the shortest cardinality of all the dominating sets in a particular graph. The domination number of G, represented by γ(G), is the cardinality of a minimum dominating set in a graph G.

The study of domination in graphs was initiated by ore who observed that for every graph G. the relation γ(G)≤ β(G) holds, where β(G), the independence number of G, is the cardinality & a largest independent set of vertices in G, Since ore‟s initial presentation, a limited amount of work has been done with dominating sets and the domination number (see [1] and [2] for surveys) Hypothetical applications of minimum dominating sets have been suggested by Berge [3] who discuss the use of the notation in devising optimal methods of radar surveillance and transmitter placement in communications networks.

That the implementations of these methods to problems modeled by large graphs may be difficult is suggested by the fact that the determination of the domination number of an arbitrary graph is an NP. Complete problem. It should be noted that bounds on γ(G) do exists Berge[4], which these bounds depend mayal so be difficult to showed that if G denotes the maximum numbere of end edges in a spanning forest of a graph G, then the sum γ(G) + β(G) equals the number of vertices in G. Other results concerning the domination number and related parameters can be found in papers by , Allan and Laskar [4] and Cockayane, Favaron, Payan and Thomsan [5], Despite the difficulty of the problem in general, the game chromatic number of various classes of graphs has received significant attention; the cases of forests [6]and planar graphs [7-11]have been studied, as have cactuses [12],and the relation to the acyclic chromatic number [13] and game Grundy number [14], to name a few avenues of research.Another significant direction of study, initiated by Bohman, Frieze, and Sudakov [15], is the study of the game chromatic number of the binomial random graph.

2. Applications of Domination in Graph

Domination in graphs has applications to several fields. Domination arises in facilitylocation problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one at- tempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a fality is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in prob- lems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

3. Definitions

Graph:AGraphGconsistsofapair(V(G),E(G))whereV(G)={v_1,v_2,…………………..v_n} is a non-empty finite set whose elements are called points or vertices or nodes, and E(G)={e_1,e_2,………..e_n} is a set of unordered pairs of distinct elements Vj of V(G) the elements of E(G) are called lines or edges of the graph G.

Loop: An edges joining vertex to itself is called a loop.

Multiple graph: If more than one line joining two vertices are allowed in a graph than it is called multiple graph. The lines joining such that two points are called multiple edges.

The degree: The degree of V is the number of edges of G incident with V count in g each loop twice.

Simplegraph: Agraph is called simple graph if it has no loops and no multiple edges.

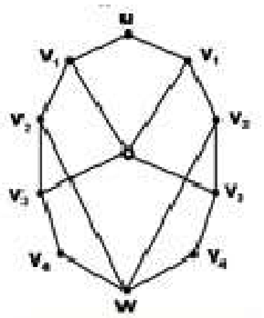
Dominating sets in graphs:-

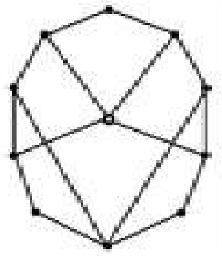
A set S $\subseteq$ V of vertices in a graph G = (V,E) is called a dominating set. If every vertex v € V is either an element of S or is adjacent to an element of S.

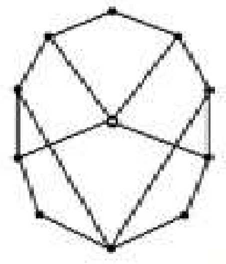
#A set S $\subseteq$ Vof vertices in a graph G= (V,E) is a dominating set. If and only if,

For every vertex v€V.S,thereexistsavertex u€Ssuch that V is adjacent to u. For every vertex u € V-S, d(r,s) ≤ 1.
N[s]=V.
For every vertex v € V-S: $|N(r) \cap S| \geq 1$.
That is every vertex v≤v-s is adjacent to at least one vertex in S.
For every vertex v € V $|N(v) \cap S| \geq 1$.
V-Sisencloveless

Example:







S_1={u, v, w} S_2={u_1, $u_2 v_1$,v_2}

Two dominating sets on a graph

Bounds on the domination number:

Bound interms of order:

An obvious upper board on the domination number is the number of the vertices in the graph. Since at least one vertex is needed to dominate a graph we have $1 \leq \gamma(G) \leq n$.for every graph of order n both these bounds are sharp.

Theorem:

Statement: If G is a graph with $\gamma_n(G)n \leq z$ then

$\gamma_n(G) \geq \gamma(G) + n\text{-}z$.

Proof: Let D be a minimum n-dominating set in G, Let u: V(G)-D, and let $v_1,v_2,\ldots\ldots v_n$ be distinct members of D that dominate u. Since D is an n-dominating set, each vertex in Yn(G)-Disdominated by atleast one member of D-($v_1,v_2,\ldots\ldots v_n$)therefore s inceuisadjacentwitheachofv_2,v_3 v_n. The set

D*=(D-{v_2,v_3 ………………………… v_n})

IsadominatingsetinG.

Hence$\gamma(G)d^{11}|D^*| = \gamma_n(G)-(n-1)+1$, So

that$\gamma_n(G) \geq \gamma(G) + n-2$.

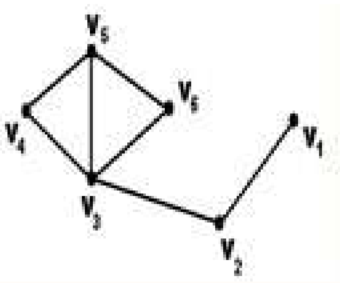
Althoughtheoremyieldsalowerboundonthedifferenceb/w$\gamma_n(G)$and$\gamma(G)$for $n \geq z$.

If often does not provide an easilycompleted lower bound on $\gamma_n(G)$ for in many cases the determination of $\gamma_n(G)$ is difficult.

Theorem:

Statement: If a graph G has no isolated vertices Then $\gamma(G) \leq n/2$.

Ex:1



Here$\gamma(G)=2, n=6$,

$n/2 = 6/2 = 3$

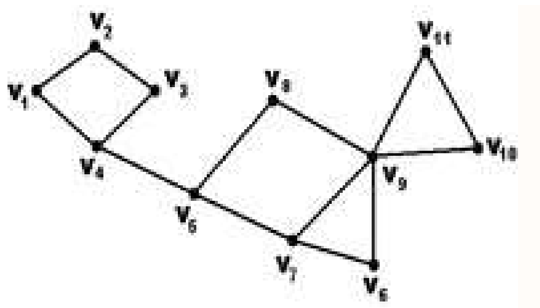
$2 \leq 3$.

Hence $\gamma(G) \leq n/2$.

Theorem:

Statement: If a graphG has $\gamma(G) \geq z$ and $\Delta(G) \leq n-\gamma(G)-1$ $m \leq$ $\frac{1}{2}$ {n- γ(G) (n- γ(G)+1}

Verification:



In a graph n=11

$\Delta(G) = 5$

m = 16

$\gamma(G)=3$

Now$\Delta(G) \leq n-\gamma(G)-1$

$\leq 11-3-1$

≤ 7

Hence$5 \leq 7$

Hence$\Delta(G) \leq n - \gamma(G) - 1$

Also$\frac{1}{2}$\{n-(G)\}\{n-γ(G)+1\}

$=\frac{1}{2}(11-3)(11-3)$

$=\frac{1}{2}(8)(9)$

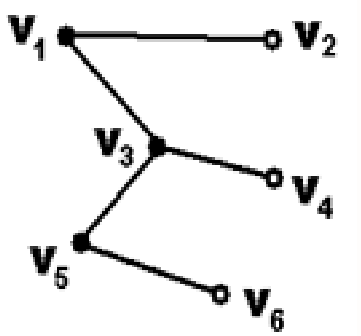
$=36$

Hence $16 \leq 36$

Hence m $\leq \frac{1}{2}$\{n-γ(G\}\{n-γ(G)+1\}

For any graph G$[n/(1+\Delta(G)] \leq \gamma(G) \leq n - \Delta(G)$

<u>Verification:</u>



Here Δ(G)=3

γ(G)=3

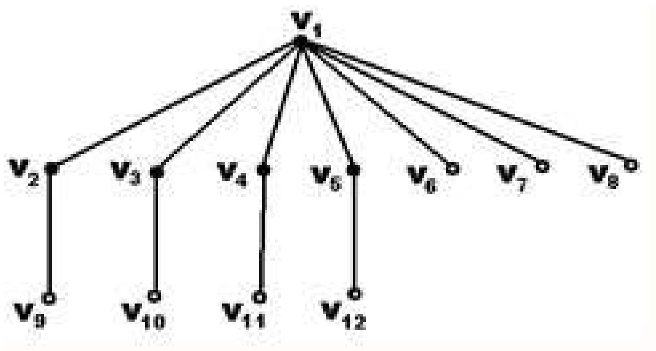
n =6

For lower bound = [n/(1+Δ(G))]=[6/(1+3)]=6/4=1.5≈2 For upper bound = n- Δ(G) = 6-3 =3

Hence2≤3≤3

Hence [n/ (1+Δ(G))]≤γ(G)≤[n– Δ(G)]

<u>Ex</u>: 2



Here n=12Δ(G)=7andγ(G) =5

For Lower bound=[n/(1+Δ(G))]=12/(1+7)=12/8=1.5≈2 For

Upper bound= n - Δ(G)= 12 - 7= 5

Hence2 ≤5≤ 5

Hence verified[n/(1+Δ(G))] ≤γ (G) ≤ n-Δ(G)

4. Conclusions

Through out this paper we have studied the relationship b/w the n-domination number and various other parameters associated with a graph we have also ventured to guess ann-dominationanaloguetoore"sclassicalresultrelatingdominationandindependencewe close with an open problem concerning the rate at which the n-domination number increases as n increases. And also The main aim of this paper is to present the importance of graph theoretical ideas in various areas of Science & Engineering for researches that they can use Domination in graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory. So, the graph theory section of each paper is given importance than to the other sections. Researches may get some information related to graph theory and its applications in various field and can get some ideas related to their field of research.

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