Static Performance of Finite Hydrodynamic Journal Bearing Operating with Lubricant Containing Nano particles under Turbulent Flow.

Duddela Siva Nanda Reddy[1], K. Thirupathi Reddy[2], G.Prasanthi[3]

[1]Research Scholar, Department of Mechanical Engineering,

Jawaharlal Nehru Technological University Anantapur, Anathapuramu. Andhra Pradesh, India.

[2]Department of Mechanical Engineering,

Rajeev Gandhi Memorial College of Engineering and Technology, Nandyal, Andhra Pradesh, India.

[3]Department of Mechanical Engineering,
Jawaharlal Nehru Technological University Anantapur, Anathapuramu. Andhra Pradesh, India.

Abstract

This research investigates the static effects of turbulence and viscosity fluctuations on a Thin film bearings. bearing, improved through the use of nano lubricants. The bearing's load-bearing capacity is directly affected by the lubricant's viscosity, which is determined by the concentration of nanoparticles in the base lubricants. To address this, a modified Reynolds equation is developed for bearing systems operating under turbulent flow conditions. The Finite Difference Method (FDM) is employed to derive a closed-form solution. Theoretical evaluations of pressure distribution and load capacity are conducted using the modified Reynolds equation across various nanoparticle concentrations. The findings reveal that incorporating nanoparticle lubricant additives significantly enhances the load capacity of the journal bearing compared to plain oils without such additives.

Keywords hydrodynamic journal bearing, Nano lubricants, turbulence, modified Reynolds equation, static performance, load capacity, friction

1. INTRODUCTION

Approximately 33% of the world's total energy is lost to friction, leading to around 70% of mechanical component failures. Reducing friction enhances machinery efficiency and significantly lowers energy consumption, which is crucial in transportation, manufacturing, and power generation. Journal bearings are widely used to support rotating shafts in machines such as steam turbines, generators, blowers, compressors, and internal combustion engines. Unlike rolling element bearings, journal bearings excel in vibration absorption, shock resistance, quiet operation, and longevity because they use a thin oil film to support the shaft[1]. Hydrodynamic journal bearings generate pressure in the lubricating oil film to support external radial loads, thereby minimizing wear and power loss. Consequently, ongoing research focuses on improving the performance of journal bearings[2].

Stable suspensions of small solid particles in a base fluid, like SAE oils, are known as nanoparticle lubricants. The lubricant's viscosity is increased and wear and friction are decreased by the inclusion of nanoparticles. It is commonly known how nanoparticle lubricant additives affect the boundary lubrication regime.
[3,4,5,6,7,8,9,10,11,12,].

The emergence of nanotechnology in the twenty-first century[13]. represents a significant technological leap. By integrating nanoparticles into lubricants, nano tribology achieves notable reductions in friction and wear while enhancing load-carrying capacity[14]. In boundary lubrication systems, nanoparticles play primary and secondary roles. In the primary effect, these nanoparticles act as ball bearings within the lubricant, reducing friction and wear between surfaces[15]. Simultaneously, in the secondary effect, nanoparticles deposit onto friction surfaces, replenishing lost mass—a phenomenon termed the mending effect—further mitigating friction and wear[16]. Additionally, nanoparticles counteract abrasive effects and minimize surface roughness, thereby boosting load-bearing capacity through a polishing effect.

Research into the effectiveness of nano lubricants in boundary lubrication has produced promising results. However, there is a noticeable lack of published studies on their application in hydrodynamic lubrication.

In this ongoing study, we employ the Krieger-Dougherty viscosity model to investigate viscosity variations, aiming to assess pressure fluctuations and load resonant capacity in hydrodynamic journal bearings.

The standard theory of lubrication states that laminar flow takes place inside lubricant films [17]. However, with high-speed machinery, this flow often becomes turbulent. Researchers have developed a number of ideas over time to investigate turbulent flow and its consequences under different lubrication conditions.

Taylor (1965) used the Ng and Pan (N-P) model and the Constantinescu model (1959) to quantitatively investigate the lubricating performance of inclined slider régimes.

[18] .While both models showed an improvement in load support compared to laminar flow, they also revealed an increase in friction force in the turbulent regime, which was considered a drawback.

Vinay Kumar offered a critical analysis of turbulent models, including the N-P model (1965) and Constantinescu's model (1959) [19].bringing attention to their ambiguity in the transitional policy.
When Reynolds numbers were higher (50,000, for example), these models gave erroneous findings because

the mean fluid inertia's substantial impact. Safar and Shawky's[20] research, however, showed that the mean fluid inertia impact was negligible up to a Reynolds number of 10,000, indicating that it may be disregarded in many real-world applications without significantly increasing inaccuracy.
Tsann-Rong [21] Lin used the N-P model to add turbulent flow conditions while examining the impact of 3-D roughness on journal bearing performance in turbulent regimes. The findings showed that raising

width, asperity height, and Reynolds number.
Researchers presented a number of low Reynolds k-ε models, including Chien [22], Launder-

Sharma [23], and Abekondah-Nagano [24], to establish turbulent flow conditions. A numerical method was presented by Lew et al. [25] to address instability in numerical solutions of equations derived from k-ε turbulence models. The different turbulent models that are currently available were briefly presented by Zhang et al. [26]. They also introduced a novel model called Zhang and Zhang's mixed k-ε model (2003), which showed better agreement with experimental results than existing models.

The Elrod and Ng model (1967) was used to accommodate turbulent flow conditions, and a bulk flow model was used to simplify the Navier-Stokes equation in Shyu et al.'s thermo-hydrodynamic analysis of finite width planar slider bearings under turbulent regimes [27]. They

supplied an empirical relationship to aid in the design for determining load support by employing the least squares method.

Frene and associates [28]. examined the effects of inertia and turbulent flow in a variety of applications, including hybrid bearings, annular seals, and concentric cylinders. In their analysis of the steady-state properties of externally adjustable single-pad fluid film bearings for both laminar and turbulent regimes, Shenoy and Pai[29] noted that radial adjustments and tilting angle could have an impact on performance metrics.

The impact of wear and recess form on capillary compensated, four-pocket, hybrid journal bearing systems operating in turbulent regimes was analytically studied by Nicodemus and Sharma [30]. They employed Constantinescu's model and the Duabrasive wear model (1983) to

include wear and turbulent circumstances, respectively, observing that bearings operating in turbulent regimes resulted in a notable reduction in fluid film stiffness.

Maneshian and Nassab [31] used computational fluid dynamics (CFD) and a numerical technique to study the thermohydrodynamic properties of infinite length journal bearings in turbulent flow conditions. A low Reynolds k-ε model was integrated to tackle the impacts of turbulent flow in journal bearings. Their results demonstrated how variables like Reynolds number, eccentricity ratio, and clearance ratio affect journal bearing performance. In a similar manner, the findings of Solghar and Nassab [33]

A dynamic analysis of short bearings was conducted by Dousti et al. [33], taking into account the effects of both inertia and turbulent flow. By incorporating convective inertia factors using an averaged velocity technique, they were able to simplify the Navier-Stokes equations. Furthermore, Constantinescu's paradigm

was used to incorporate turbulence terms into the equation that governs pressure. Their results demonstrated how much more important turbulence is than inertia effects. Unexplored are studies on textured surfaces despite the wealth of research on turbulent flow in the hydrodynamic lubrication of smooth bearings.

Therefore, the current study aims to bridge this gap by examining textured parallel sliding contacts under turbulent flow conditions.

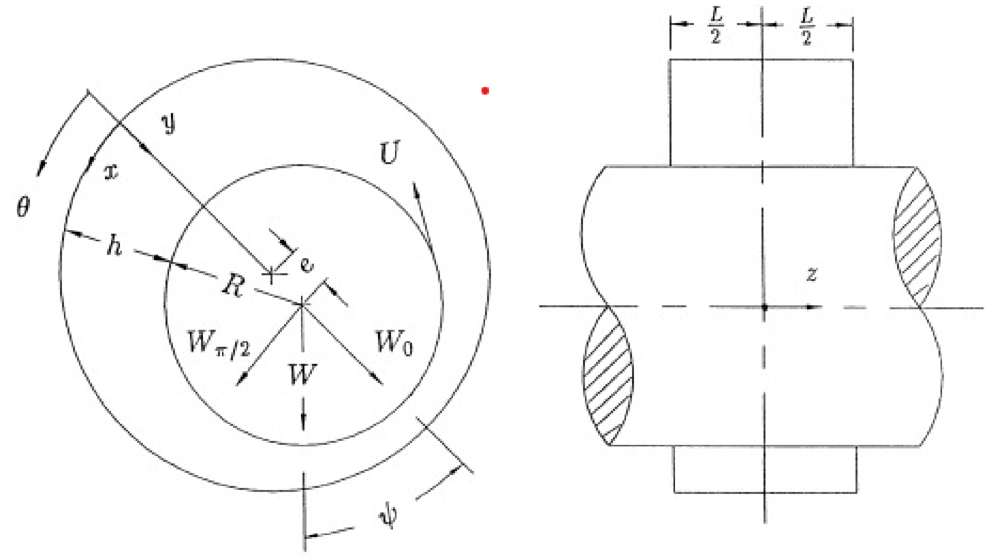

Fig. 1. Physical configuration of a finite journal bearing.

2. THEORETICALINVESTIGATION

The pressure equation for a journal bearing was considered as:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6U\eta\frac{dh}{dx} \qquad [1]$$

It is used for conclusion the pressure dispersal in journal bearing**.**

The governing equation non-dimensional form as

$$\frac{\partial}{\partial \theta}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial \bar{z}}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \bar{z}}\right)=6\bar{\mu}\frac{d\bar{h}}{d\theta} \qquad [2]$$

2.1 Delimitations

The Reynolds Equation is classified as an elliptical partial differential equation, and solving it calls for a method based on boundary value issues.

At = 0 0 and = 360 0 P = 0
At Z = + L/2, -L/2, P = 0.
where Z is the bearing axis eccentric to the shaft axis and is the circumferential angle.

2.1 The current research solves the turbulent Reynolds equation for an incompressible flow given by using the N-P model. [18].

$$\frac{\partial}{\partial x}\left(\frac{h^3}{k_x\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{k_z\eta}\frac{\partial p}{\partial z}\right)=\frac{u}{2}\frac{\partial h}{\partial x}$$

The terms kxk_xkx and kzk_zkz represent frictional flow rate parameters in the x- and z-directions, respectively. Ng and Pan [39] graphically depicted these parameters. Later, Taylor [18] proposed empirical relations for the kxk_xkx and kzk_zkz terms using curve fitting techniques as follows:

$$k_x = 12 + C_x \mathrm{Re}^{n_x}$$
$$k_z = 12 + C_z \mathrm{Re}^{n_z}$$

where Cx, Cz, nx and nz are constants with dependence on Reynolds number.

3.1 Assessment of Nano fluid Viscosity: Viscosity Models: Estimation of the Nanofluid viscosities:

There are certain theoretical formulas used to find the viscosities of Nanofluid.

(i) Einstein Model [34]

$\mu_{nf} = \mu_{bf}\,(1 + 2.5\,\phi)$

$\bar{\mu}$=μn**T** μb**T**

= (1 + 2.5 ϕ) (3)
where Vp is the nanoparticals' volumetric concentration. When $\phi \leq 0.02$, Einstein's formula can be applied.
(ii) For a moderate particle concentration, Brickman [35] expanded the formula as follows: $\bar{\mu}$=μnf • bf = 1 (1−ϕ) 2.5 (4)
(iii) Batchelor [36] developed a model that took into account the Brownian motion of the fluid's particles: $\bar{\mu}$=μnf • bf = (1 + 2.5 ϕ + 6.5 ϕ 2) (5)
(iv) Kole and Dey [37] investigated the viscosity fluctuation in gear oil using CuO nanoparticles.
The research 1 2 2 2 9 9 9
It was found that viscosities could be simulated using a modified Krieger-Dougherty viscosity model, with outcomes that closely matched experimental observations.

$$\bar{\mu} = \frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\phi}{\phi_m}\right)^{-[\eta]\phi_m} \qquad (6)$$

The maximal particle packing fraction, or ϕ_m, is roughly 0.605, while the intrinsic viscosity, or [η], is defined by Kole and Dey as 2.5. The aforementioned equation was modified to include the packing fraction inside the aggregate structure of nanoparticles. The Krieger-Dougherty equation is altered as a result of this modification.

$$\bar{\mu} = \frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\phi_a}{\phi_m}\right)^{-[\eta]\phi_m} \quad (7)$$

$\emptyset_a = \emptyset \left(\frac{a_a}{a}\right)^{3-D}$ Value of D is 1.8 for and $\phi_m = 0.605$ for Nanofluids.

$\frac{a_a}{a} = 7.77$ for TiO_2 based nano-lubricant.

4.BEARINGSTATICCHARACTERISTICS.

The load-carrying capacity is calculated by integrating the ®lm pressure acting on the journal surface. Along and perpendicular to the line of centres,

The component loads are expressed as follows:

$W_r = WCos\emptyset = -2\int_0^{\pi}\int_{Z=0}^{Z=1/2} PRCos\theta d\theta dz$

$W_t = Wsin\emptyset = 2L\int_0^{\pi}\int_{Z=0}^{Z=1/2} PRSin\theta d\theta dz$

where, *Wr* and *Wt* are the components of load carrying capacity along and tangential to the line of centres respectively.

Journal bearing load carrying capacity in non-dimensional form [9] as:

$\overline{W} = \frac{WC^2}{\mu_{bl}UL^3} = \sqrt{(\overline{w}_r)2 + (\overline{w}_t)^2}$

4.2. **Attitude angle***:* It is the angle between the line passing through the centres and the load acting direction, and is given by

$\alpha = \tan^{-1}\left[\frac{\overline{W_t}}{\overline{W_r}}\right]$

$\alpha = \text{ArcTan}\frac{\pi}{4} * \frac{(1-\epsilon^2)^{1/2}}{\epsilon}$

4.3.Abrasion Force

The resistance force is obtained by adding shear stress transversely the bearing area. The resistanceforce non-dimensional form as:

$\bar{F} = FC/\mu_{bl}URL$

$\bar{F} = 2R\int_{z=0}^{z=1/2}\int_0^{\pi} \tau d\theta dz$

$$\bar{F} = (1 - \frac{\phi}{\phi_m})^{-\eta*\phi_m} * \frac{2*\pi}{(1-\epsilon^2)^{1/2}}$$

The friction parameter is then obtained as:

$$F\frac{R}{C} = \frac{2*\pi}{\bar{w}[1-\epsilon^2]^{1/2}}$$

$$\bar{F} = (1 - \frac{\phi_a}{\phi_m})^{-\eta*\phi_m} * \frac{2*\pi^2}{(1-\epsilon^2)^{1/2}}$$

SOLUTION PROCEDURE:

The solution to the Reynolds equation employs the finite difference method to compute the pressure distribution around the journal. This method adopts an iterative approach where, initially, all pressures at each point are set to zero. In each subsequent iteration, the pressure computed from the previous iteration is used to update the pressure values instead of zero. This iterative process continues until the change in pressure drops below one-thousandth of the last pressure value, known as epsilon (ε), ensuring accuracy within computational constraints. Once this criterion is met, indicating minimal further change in pressure, the iterative procedure concludes.

5.RESULTS AND DISCUSSION

In this study, the authors utilize the improved Krieger-Dougherty viscosity model to investigate variations in viscosity, pressure distribution, and load carrying capacity. They examine nanoparticle concentrations ranging from 0.5 to 2.5 Vol % and eccentricity ratios (ε) from 0.1 to 0.9 as key parameters. The findings are presented graphically, with Figure 3 illustrating the dimensionless load carrying capacity (W) plotted against eccentricity ratio (ε) for various nanoparticle size fractions.

Their analysis reveals that incorporating nanoparticles significantly enhances film pressure, particularly notable at higher eccentricity ratios. Furthermore, they observe improvements in load carrying capacity and friction force, coupled with a reduction in the coefficient of friction, especially pronounced when using 2.5% vol nanoparticles compared to base fluids.

Figure 4 highlights that turbulent flow conditions result in higher load carrying capacity and friction force, accompanied by a lower coefficient of friction compared to laminar flow. Additionally, Figure 5 shows that the attitude angle decreases as the eccentricity ratio increases for both base and nano lubricants. Interestingly, the addition of nanoparticles does not appear to significantly alter this relationship..

Similarly, Figures 6 and 7 illustrate how the eccentricity ratio influences friction for both base and Nano lubricants, show casing a descend in the coefficient of friction under turbulent flow conditions relative to laminar flow.

Figure 8 delves into the dimensionless load-carrying power versus eccentricity ratio, revealing that nano fluid effects augment load-carrying capacity, particularly in high eccentricity ratios and turbulent flow conditions, especially in longer bearings.

Furthermore, Figure 9 delves into the attitude angle versus eccentricity ratio parameter, indicating a decrease in attitude angle with increasing eccentricity ratio, with no significant impact from nanoparticle addition.

Lastly, Figure 10 explores Friction versus eccentricity ratio for different nanoparticle size fractions under turbulent flow conditions, showcasing reduced friction with nanoparticle addition, especially at high eccentricity ratios and turbulent flow conditions, with more pronounced effects in longer bearings.

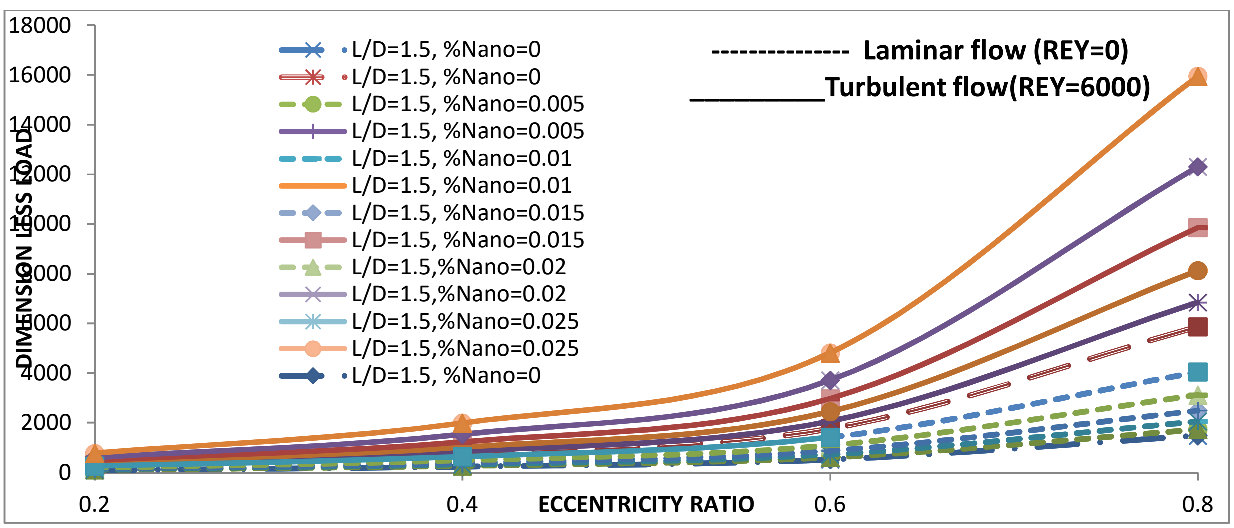

Fig 3.Dimensionless load capacity W vs. Eccentricity ratio with λ=1.5 at different volume fractions of nano fluid.

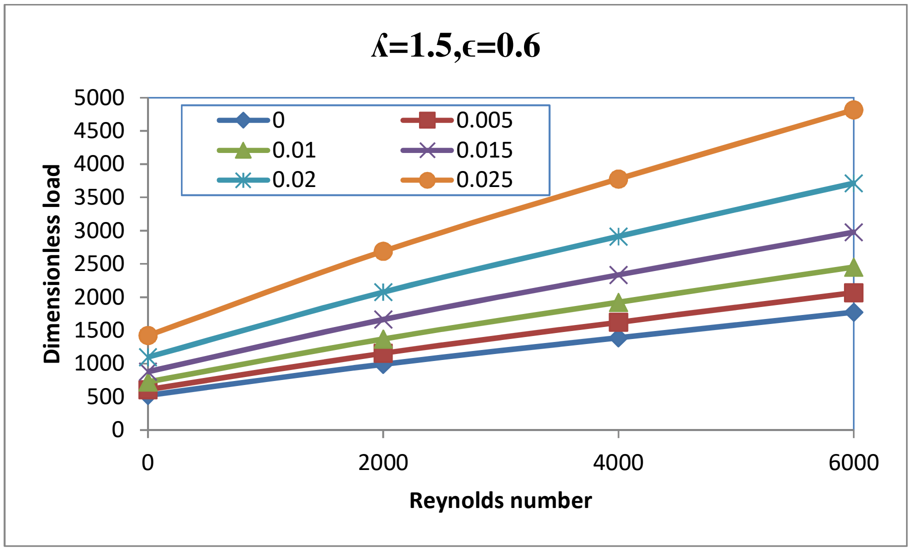

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Fig 4.Dimensionless load capacity W vs. Reynolds number with ʎ=1.5 at different volume fractions of nano fluid.

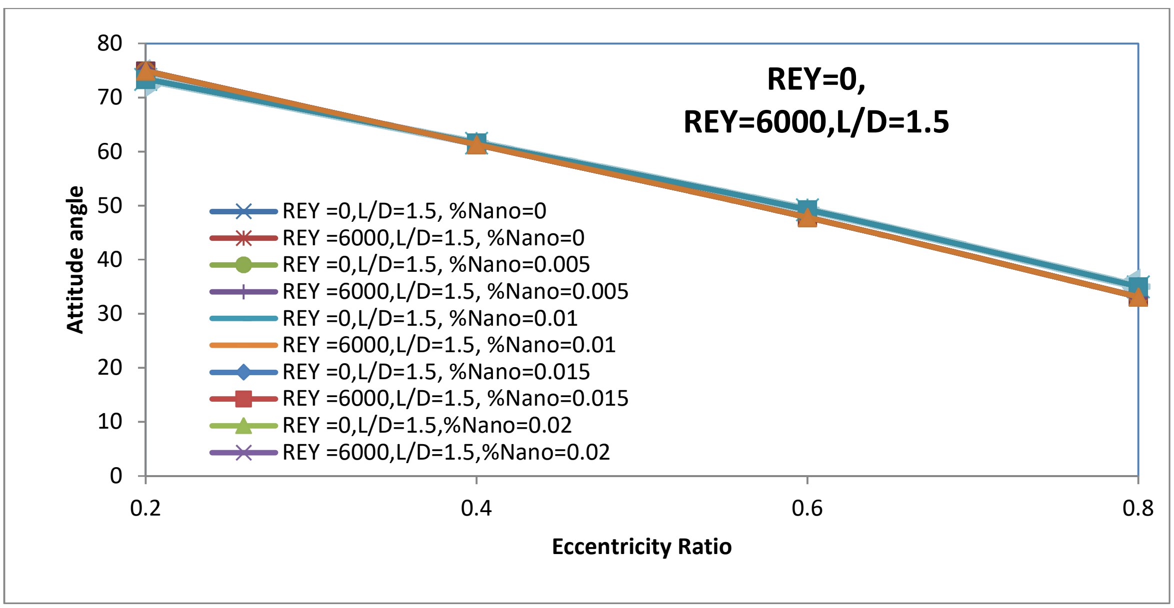

Fig 5. Attitude angle vs. Eccentricity ratio with ʎ=1.5 at different volume fractions of nano fluid.

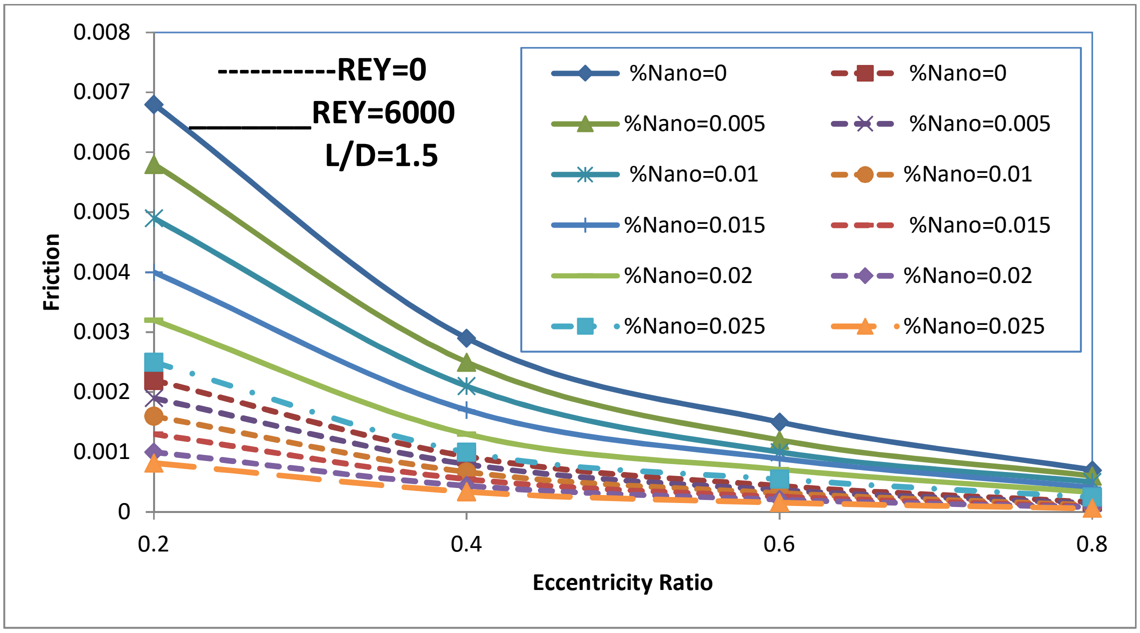

Fig 6. Friction vs. Eccentricity ratio with ʎ=1.5 at different volume fractions of nano fluid.

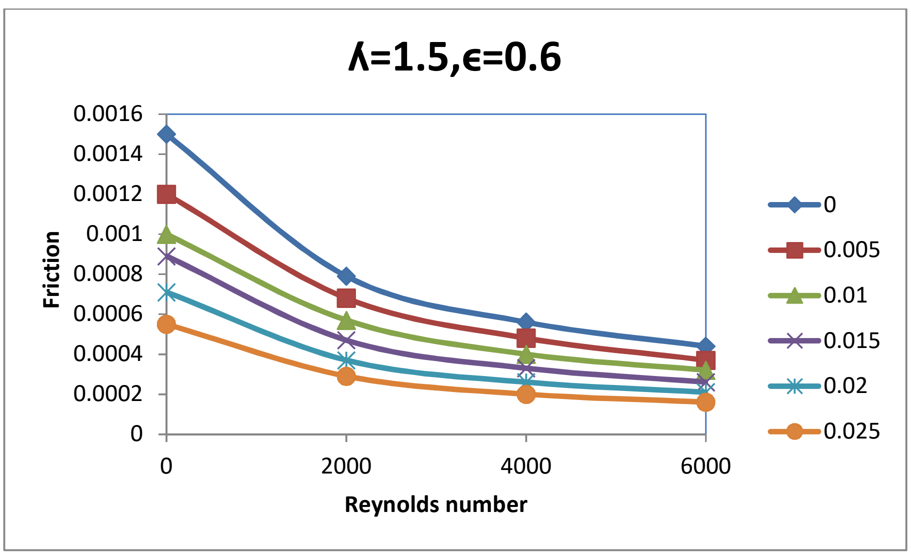

Fig 7.Friction vs. Reynolds number with ʎ=1.5 at different volume fractions of nano fluids.

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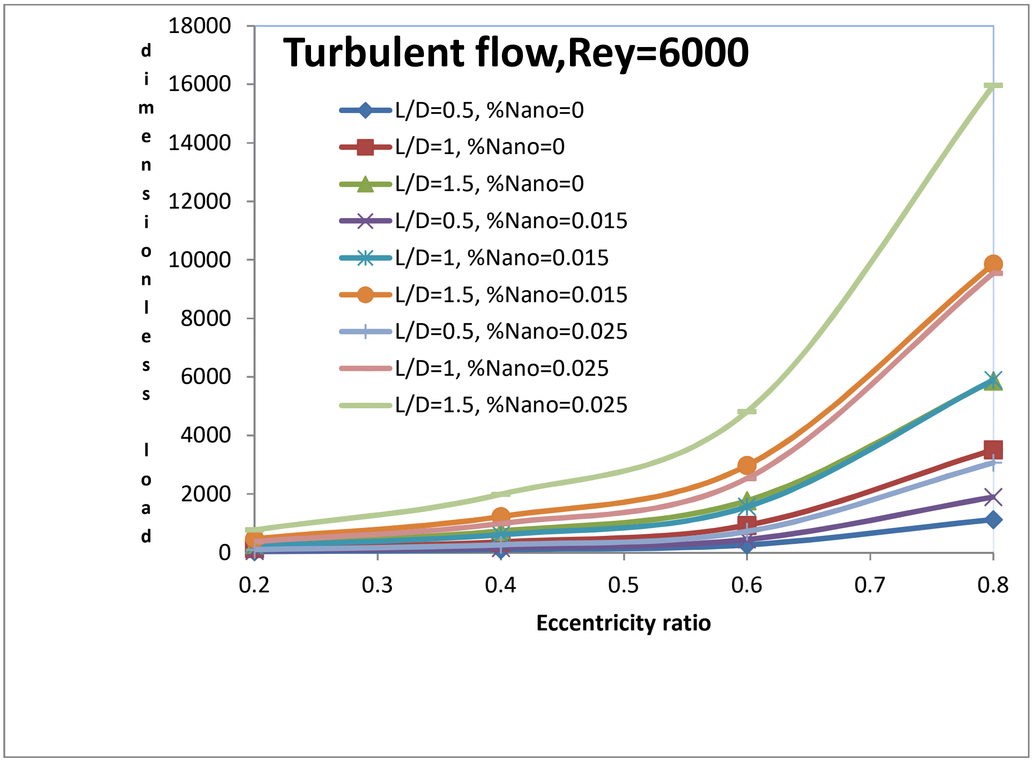

. Fig 8. Dimensionless load capacity vs. Eccentricity ratio for different λ at different volume fractions.

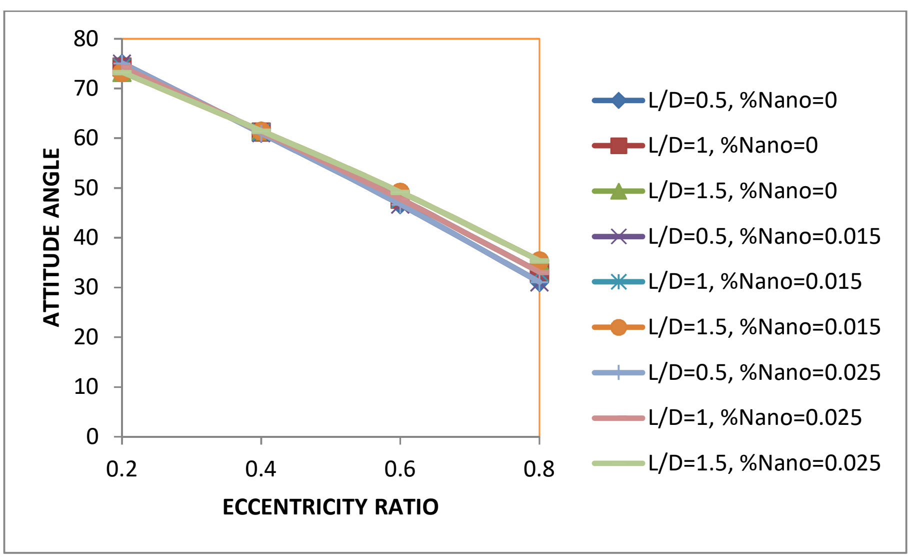

Fig 9. Attitude angle vs. Eccentricity ratio for different λ at different volume fractions of nano fluids.

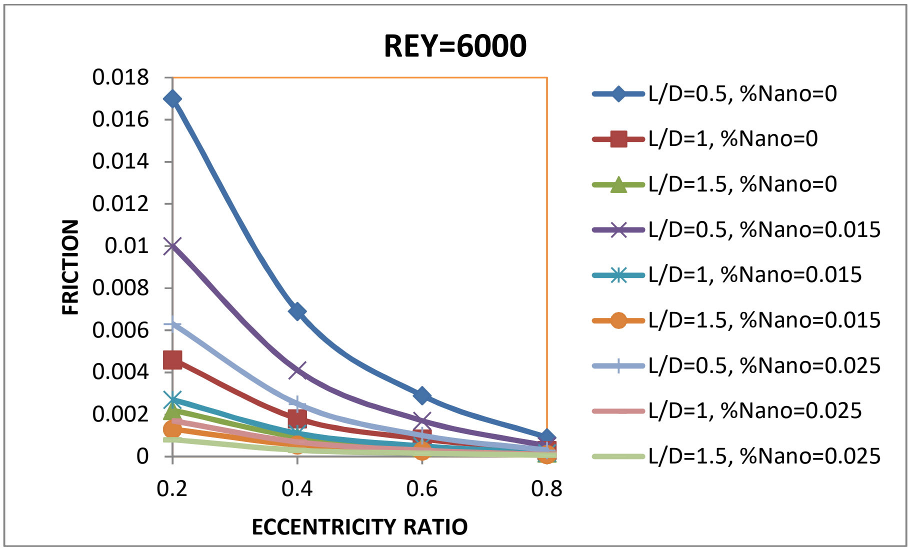

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Fig 10. Friction vs. Eccentricity ratio for different λ at different volume fractions of nano fluid.

6. CONCLUSION

In this study, the Stokes micro continuum theory is employed to investigate how nano lubricants affect the static characteristics of finite journal bearings. The modified Reynolds equation, which integrates viscosity effects from a base lubricant mixed with additives, is derived using the Stokes constitutive equations. Numerical iteration and the consideration of Reynolds boundary conditions are employed to calculate the pressure distribution, enabling the determination of crucial bearing properties.

The findings highlight the substantial influence of nano lubricants in the system, often outweighing the effects of the base lubricant. As the nano parameter approaches zero, the behavior of the system aligns more closely with that observed in the classical base-lubricant scenario.

Key conclusions drawn from the study include:

1.Nano fluids play a significant role in influencing bearing characteristics, underscoring their importance in lubrication systems.

2.Nano lubricants enhance load-carrying capacity and decrease attitude angle and friction parameters compared to base-lubricant scenarios.

3.These advantages are especially pronounced in bearings operating at higher eccentricity ratios.

4.The inclusion of nano lubricants blended with additives yields beneficial results in terms of improving load capacity and reducing friction in lubrication systems.

Overall, the study highlights the crucial role of nano lubricants in enhancing the overall performance of lubrication systems, particularly in terms of increasing load capacity and decreasing friction.

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Nomenclature

r	Radial clearance (m)	
e	Eccentricity (m)	
h	Fluid film thickness (m)	$h = (C + e\cos\theta)$
Ɛ	Eccentricity ratio,	$\varepsilon = e/C$
ħ	Fluid film thickness in Non-Dimensional form (m)	$\bar{h} = h/c = (1 + \varepsilon\cos\theta)$
L	Bearing length (m)	
P	Lubricant fluid film pressure (N/m^2) p	
P	Fluid film Pressure in Non-Dimensional form $\bar{P} = PC^2/\mu_{bl}UR$	
R	Journal Radius (m)	
U	Surface Velocity of Journal (m/s) U	
W_r	Radial component of load carrying capacity (N),	
W_t	Tangential component of load carrying capacity (N),	
W	Total Load (N)	
W	Non Dimensional Load Carrying Capacity, $\overline{W} = WC^2/\mu_{bl}UL^3$	

α□	Attitude angle (rad)
F	Friction force (N)
F	Non-dimensional friction force,(N) F = FC/μ_{bl}URL
S	Somerfield number
x	Bearing coordinates in the circumferential direction (m), $x = R\theta$
z	Bearing coordinates in the axial direction (m), $z = \bar{z}L$
μ_{base}	Viscosity of base oil (Pa-s)
μ_{nano}	Viscosity of the Nano lubricant (Pa-s)
μ	Relative Viscosity in Non-Dimensional form
Φ	Volume fraction of Nanoparticles
Φm	Maximum particle packing fraction
θ	Angular Coordinates (rad)
ω	Angular Velocity of Journal(rad/s)
η	Intrinsic viscosity
a	Radii of primary Nano particles(nm)
a_a	Radii of aggregate Nano particles(nm)
D	fractal index.
ʎ	L/D Ratio.