"The Mohand Transform on $\mathbb{R}^n$"

A. M. Mahajan, N. S. Pimpale

Department of Mathematics Walchand College of Arts and Science, Solapur.

Department of Mathematics Rajarshi Shahu College, Latur.

Abstract: In this paper the 'n- dimensional Mohand transform' is developed with the help of Mohand transform. Some properties of n- dimensional Mohand transforms are also given.

Keywords: Mohand Transform, Integral Transform, Convolution.

Introduction:

The study of n- dimensional integral transform became a tool to develop the technique of extending certain integral transform. The n- dimensional Mohand transform is developed on the same step. The new integral transform Mohand transform [1] was developed by Mohand M. The transform technique is used in 18th century to solve differential equations which are difficult to solve by ordinary sense. The Kamal transform, Elzaki, Aboodh, Mahgoub, Sumudu, Laplace [3], [4], [5] are defined in literature. Many researchers used these integral transforms in various fields. These transforms have been developed by various researchers to solve integro differential equation, partial differential equation, boundary value problems, signal processing, ordinary and simultaneous differential equation. Also Kilicman and Gadain [9] gave application of some integral transform.

The new integral transform Mohand transform is defined for function of exponential order in the set A defined by

$$A = \left\{ f(t) : \exists\, M, K_1, K_2 > 0, |f(t)| < Me^{\frac{|t|}{K_j}}, t \in (-1)^j [0, \infty] \right\} \quad \_(1)$$

$$M\{f(t)\} = m(z) = z^2 \int_0^\infty f(t)\, e^{-zt} dt\,, t \geq 0, K_1 \leq z \leq K_2 \quad \_(2)$$

$$M\{f(x_1, x_2, \dots, x_n)\} = f(p_1, p_2, \dots, p_n) =$$
$$p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty f(x_1, x_2, \dots, x_n) e^{-(p_1x_1 + p_2x_2 + \dots + p_nx_n)}\, dx_1 dx_2 \dots dx_n \quad \_(3)$$

In this section we find Mohand transform on $\mathbb{R}^n$ of simple functions.

Property (i) $f(x_1, x_2, \dots, x_n) = 1$ then $M\{1\} = f(p_1, p_2, \dots, p_n) = p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty 1 \cdot e^{-(p_1x_1 + p_2x_2 + \dots + p_nx_n)}\, dx_1 dx_2 \dots dx_n$

$$= p_1{}^2 p_2{}^2 \dots p_n{}^2 \cdot \frac{1}{p_1} \cdot \frac{1}{p_2} \cdots \frac{1}{p_n}$$

$$= p_1 \cdot p_2 \cdot \dots \cdot p_n$$

Property (ii) $f(x_1, x_2, \dots, x_n) = x_1 \cdot x_2 \cdot \dots \cdot x_n$, then $p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty x_1 \cdot x_2 \cdot \dots \cdot x_n \cdot e^{-(p_1x_1 + p_2x_2 + \dots + p_nx_n)}\, dx_1 dx_2 \dots dx_n$

Integrating by parts, we get, $M\{f(x_1, x_2, \dots, x_n)\} = 1$

Property (iii) $M\{e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}\} = \frac{p_1{}^2 p_2{}^2 \dots p_n{}^2}{(p_1-a_1)(p_1-a_2)\cdots(p_1-a_n)}$

Proof: $M\{e^{\alpha_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n}\}$

$$= p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{a_1 x_1+\alpha_2 x_2+\cdots+\alpha_n x_n} \cdot e^{-(p_1 x_1+p_2 x_2+\cdots+p_n x_n)}\, dx_1 dx_2 \dots dx_n$$

$$= p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(p_1-\alpha_1)x_1-(p_2-\alpha_2)x_2-\cdots-(p_n-\alpha_2)x_n}\, dx_1 dx_2 \dots dx_n$$

$$= \frac{p_1{}^2 p_2{}^2 \dots p_n{}^2}{(p_1-a_1)(p_1-a_1)\cdots(p_1-a_n)}$$

Property (iv) Linearity Property:

$$M\{c_1 f_1(x_1,x_2,\dots,x_n) + c_2 f_2(x_1,x_2,\dots,x_n) + \cdots + c_n f_n(x_1,x_2,\dots,x_n)\}$$
$$= c_1 M\{f_1(x_1,x_2,\dots,x_n)\} + c_2 M\{f_2(x_1,x_2,\dots,x_n)\} + \cdots + c_n M\{f_n(x_1,x_2,\dots,x_n)\}$$

Proof: $M\{c_1 f_1(x_1,x_2,\dots,x_n) + c_2 f_2(x_1,x_2,\dots,x_n)\}$

$$= p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} \{c_1 f_1(x_1,x_2,\dots,x_n) + c_2 f_2(x_1,x_2,\dots,x_n) + \cdots + c_n f_n(x_1,x_2,\dots,x_n)\}\, dx_1 dx_2 \dots dx_n$$

$$= p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} \{c_1 f_1(x_1,x_2,\dots,x_n)\}\, dx_1 dx_2 \dots dx_n$$
$$+ p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} \{c_2 f_2(x_1,x_2,\dots,x_n)\}\, dx_1 dx_2 \dots dx_n$$
$$+ \cdots + p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} \{c_n f_n(x_1,x_2,\dots,x_n)\}\, dx_1 dx_2 \dots dx_n$$

$$= c_1 p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} f_1(x_1,x_2,\dots,x_n)\, dx_1 dx_2 \dots dx_n$$
$$+ c_2 p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} f_2(x_1,x_2,\dots,x_n)\, dx_1 dx_2 \dots dx_n + \cdots$$
$$+ c_n p_1{}^2 p_2{}^2 \dots p_n{}^2 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-p_1 x_1-p_2 x_2-\cdots-p_n x_n} f_n(x_1,x_2,\dots,x_n)\, dx_1 dx_2 \dots dx_n$$
$$= c_1 M\,\{f_1(x_1,x_2,\dots,x_n)\} + c_2 M\,\{f_2(x_1,x_2,\dots,x_n)\} + \cdots + c_n M\,\{f_n(x_1,x_2,\dots,x_n)\}$$

Definition: If $f(x_1, x_2, \dots, x_n)$ and $g(x_1, x_2, \dots, x_n)$ are integrable functions then convolution of $f(x_1, x_2, \dots, x_n)$ and $g(x_1, x_2, \dots, x_n)$ is given by

$(f ** g)(x_1, x_2, \dots, x_n) =$

$\int_0^{x_1}\int_0^{x_2} \dots \int_0^{x_n} f(\alpha_1, \alpha_2, \dots, \alpha_n)\, g(x_1-\alpha_1, x_2-\alpha_2, \dots, x_n-\alpha_n) d\alpha_1\, d\alpha_2 \dots d\alpha_n$ and ** denotes the convolution with respect to $x_1, x_2, \dots, x_n$.

Property 5 (Shifting Property): Let $f(x_1, x_2, \dots, x_n)$ be a continuous function and $M[f(x_1, x_2, \dots, x_n)] = f(p_1, p_2, \dots, p_n)$ then

$$M[e^{\alpha_1 x_1+\alpha_2 x_2+\dots+\alpha_n x_n} f(x_1, x_2, \dots, x_n)] = \frac{p_1 p_2 \dots p_n}{(p_1-\alpha_1)(p_2-\alpha_2)\dots(p_n-\alpha_n)} f\big((p_1-\alpha_1), (p_2-\alpha_2), \dots, (p_n-\alpha_n)\big)$$

Proof: From the definition of n dimensional Mohand transform,

$$M[e^{\alpha_1 x_1+\alpha_2 x_2+\dots+\alpha_n x_n} f(x_1, x_2, \dots, x_n)] = {p_1}^2 {p_2}^2 \dots {p_n}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{\alpha_1 x_1+\alpha_2 x_2+\dots+\alpha_n x_n} \cdot e^{-(p_1 x_1+p_2 x_2+\dots+p_n x_n)} f(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n$$

$$= {p_1}^2 {p_2}^2 \dots {p_n}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(p_1-\alpha_1)x_1-(p_2-\alpha_2)x_2-\dots-(p_n-\alpha_2)x_n} [f(x_1, x_2, \dots, x_n)]\; dx_1 dx_2 \dots dx_n$$

$$= {p_1}^2 {p_2}^2 \dots {p_n}^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(p_1-\alpha_1)x_1} e^{-(p_2-\alpha_2)x_2} \dots e^{-(p_n-\alpha_n)x_n} [f(x_1, x_2, \dots, x_n)]\; dx_1 dx_2 \dots dx_n$$

$$= \frac{p_1 p_2 \dots p_n}{(p_1-\alpha_1)(p_2-\alpha_2)\dots(p_n-\alpha_n)} f\big((p_1-\alpha_1), (p_2-\alpha_2), \dots, (p_n-\alpha_n)\big)$$

Property 6 (Periodic Function): Let $M[f(x_1, x_2, \dots, x_n)]$ exist, where $f(x_1, x_2, \dots, x_n)$ describes a periodic function of periods $\alpha_1, \alpha_2, \dots, \alpha_n$ such that

$f(x_1+\alpha_1, x_2+\alpha_2, \dots, x_n+\alpha_n) = f(x_1, x_2, \dots, x_n), \forall x_i, i = 1, 2, \dots, n$ then,

$$M[f(x_1, x_2, \dots, x_n)] = \frac{1}{(1-e^{-(p_1\alpha_1+p_2\alpha_2+\dots+p_n\alpha_n)})}\left({p_1}^2 {p_2}^2 \dots {p_n}^2 \int_0^{\alpha_1}\int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1 x_1+p_2 x_2+\dots+p_n x_n)} [f(x_1, x_2, \dots, x_n)]\; dx_1 dx_2 \dots dx_n\right)$$

Proof: By definition of n dimensional Mohand transform,

$$M[f(x_1,x_2,\dots,x_n)] = p_1^2 p_2^2 \dots p_n^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$

$$= p_1^2 p_2^2 \dots p_n^2 \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1x_1+px_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n + p_1^2 p_2^2 \dots p_n^2 \int_{\alpha_1}^\infty \int_{\alpha_2}^\infty \dots \int_{\alpha_n}^\infty e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$

Put $x_1=\alpha_1+r_1, x_2=\alpha_2+r_2,\dots,x_n=\alpha_n+r_n$

$$= p_1^2 p_2^2 \dots p_n^2 \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n + p_1^2 p_2^2 \dots p_n^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(p_1(\alpha_1+r_1)+p_2(\alpha_2+r_2)+\dots+p_n(\alpha_n+r_n))} f\big((\alpha_1+r_1),(\alpha_2+r_2),\dots,(\alpha_n+r_n)\big)\ dr_1dr_2\dots dr_n$$

Using the periodicity of the function $f(x_1,x_2,\dots,x_n)$ above equation can be written as

$$f(p_1,p_2,\dots,p_n) = p_1^2 p_2^2 \dots p_n^2 \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n + e^{-(p_1\alpha_1+p_2\alpha_2+\dots+p_n\alpha_n)} p_1^2 p_2^2 \dots p_n^2 \int_0^\infty \int_0^\infty \dots \int_0^\infty e^{-(p_1r_1+p_2r_2+\dots+p_nr_n)} f(r_1,r_2,\dots,r_n)\ dr_1dr_2\dots dr_n$$

$$f(p_1,p_2,\dots,p_n) = p_1^2 p_2^2 \dots p_n^2 \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n + e^{-(p_1\alpha_1+p_2\alpha_2+\dots+p_n\alpha_n)} f(p_1,p_2,\dots,p_n)$$

$$f(p_1,p_2,\dots,p_n)[1-e^{p_1\alpha_1+p_2\alpha_2+\dots+p_n\alpha_n}] = p_1^2 p_2^2 \dots p_n^2 \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} f(x_1,x_2,\dots,x_n)\ dx_1dx_2\dots dx_n$$

$$f(p_1,p_2,\dots,p_n) = \frac{1}{(1-e^{-(p_1\alpha_1+p_2\alpha_2+\dots+p_n\alpha_n)})}\left(p_1^2 p_2^2 \dots p_n^2 \int_0^{\alpha_1} \int_0^{\alpha_2} \dots \int_0^{\alpha_n} e^{-(p_1x_1+p_2x_2+\dots+p_nx_n)} [f(x_1,x_2,\dots,x_n)]\ dx_1dx_2\dots dx_n\right)$$

References:

[1]. Mohand M., Mahgoub A, The New Integral Transform “Mohand Transform”, Advances in Theoretical and Applied Mathematics ISSN 0973-4554 Volume 12, (2017), No. 2, 113-120

[2]. A. Kilicman, M Omran, On double Natural transform and its application. Journal of Nonlinear Sciences and Applications, Vol. 10 Issue 4, (2017), 1744-1754.

[3]. Kamal A., Sedeeg H., the New integral transform “Kamal Transform” Advances in theoretical and Applied Mathematics Vol. 11 (2016), 451-458.

[4]. T. M. Elzaki, The New Integral Transform “Elzaki Transform”, Global Journal of Pure and Applied Mathematics, Vol. 7 No.1 (2011), 57-64

[5]. Aboodh K. S., The New Integral Transform “Aboodh Transform”, Global Journal of Pure and Applied Mathematics, 9(1), (2013), 35-43.

[6]. Mohand M. Mahgoub A, The New Integral Transform “Mahgoub Transform” Advances in Theoretical and Applied Mathematics Volume 11, No.4, (2016), 391-398.

[7]. WatugalaG. K., Sumudu transform: a integral transform to solve differential equations and control engineering problems, International Journal of Mathematical Education in Sciences and Technology, Vol. 24, Issue 1, (2006), 35-43.