DIVISOR CORDIAL LABELING IN STAR RELATED GRAPH

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ABSTRACT

Cahit[3] have introduced cordial labeling in 1987. Let G be a simple graph with p vertices and q edges. A vertex labeling function is defined as S: δ(G) → $\{0,1\}$ and an induced edge labeling function S^*: β(G) → $\{0,1\}$ defined by $S^*(bc) = |s(b) - s(c)|$, ∀ $bc \in$ β(G) satisfies$|\delta_{S^*}(0) - \delta_{S^*}(1)| \leq 1$and $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$ is said to be a cordial labeling. Varatharajan.et.al.[10] have introduced divisor cordial labeling as a variant of cordial labeling. In this paper, we investigate the existence of some divisor cordial labelings for Bistar related graph.

keywords: Graph labeling, Star graph, Cordial labeling, Divisor cordial labeling.

1. INTRODUCTION

In 1967[3], Rosa introduced the concept of graph labeling. Assigning an integer to the edges or vertices or to both on certain conditions is said to be a graph labeling. In 2023[2], Bala .et.al. introduced the concept of Extended triplicate graph of star ETG($k_{1,p}$) .In 2011[10],Varatharajan.et.al., introduced the concept of divisor cordial labeling. Let $G = (\delta(G), \beta(G))$ be a graph with p vertices and q edges. A bijective function S : δ (G) → {1,2,3,….,p}is said to be a divisor cordial labeling. If an induced function S*(bc) $=\begin{cases} 1 & (s(b)|s(c))\ or (s(c)|s(b)) \\ 0 & otherwise \end{cases}$, ∀ bc ∈ β(G) satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}\ (1)| \leq 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.

The concept of Sum divisor cordial labeling was introduced by Lourdusamy.et.al.,[7]. Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a sum divisor cordial labeling. If an induced function S^*: β(G) → $\{0,1\}$ defined by $S^*(bc) = \begin{cases} 1 & ;\ if (2|s(b) + s(c)) \\ 0 & ;\ otherwise \end{cases}$, ∀ bc ∈ β(G) satisfies$|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a Sum divisor cordial labeling is called as sum divisor cordial graph.

The concept of Subtract divisor cordial labeling and multiply divisor cordial labeling was introduced by J.T.Gondalia.et.al.,[5, 6]. Let $G = \left(\delta(G), \beta(G)\right)$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a subtract divisor cordial labeling. If an induced function S^*: β (G) → $\{0,1\}$ defined by $S^*(bc) = \begin{cases} 1 & ;\ if\ (2|s(b) - s(c)) \\ 0 & ;\ otherwise \end{cases}$,∀ bc ∈ β(G)satisfies $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a Subtract divisor cordial labeling is called as subtract divisor cordial graph. Let $G = \left(\delta(G), \beta(G)\right)$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a multiply divisor cordial labeling. If an induced function S^*: β(G) → $\{0,1\}$ defined by $S^*(bc) = \begin{cases} 1 & ;\ if (2|s(b). s(c)) \\ 0 & ;\ otherwise \end{cases}$, ∀ bc ∈ β(G) satisfies $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a Multiply divisor cordial labeling is called as Multiply divisor cordial graph.

The concept of Square divisor cordial labeling was introduced by Murugesan[9].Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A bijective function S: $\delta(G) \rightarrow \{1, 2, 3, \ldots\ldots, p\}$ is said to be a square divisor cordial labeling. If an induced function $S^*: \beta(G) \rightarrow \{0,1\}$ defined by $S^*(bc) = \begin{cases} 1; & if((s(b))^2|s(c))or(s(b)|(s(c))^2) \\ 0; & otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ satisfies

$|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a square divisor cordial labeling is called as square divisor cordial graph

Stimulation from the above studies, In this paper we investigate the existence of Divisor cordial labeling, Square divisor cordial labeling, Sum divisor cordial labeling, Subtract divisor cordial labeling, and Multiply divisor cordial labeling in the context of Extended Triplicate graph of bistar.

2. PRELIMINARIES

In this section, we discuss about the basic notions related to this paper.

Definition 2.1[2]: : Let G be a bistar graph $B_{(p,l)}$ .The triplicate of bistar graph with the vertex set $\delta'(G)$and edge set $\beta'(G)$is given by $\delta'(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_j \cup d_j' \cup d_j'' / 1 \leq i \leq p, 1 \leq j \leq l\}$ and $\beta'(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_j' \cup b_1''d_j' \cup b_1'd_j \cup b_1'd_j'' / 1 \leq i \leq p, 1 \leq j \leq l\}$. Clearly, Triplicate of bistar graph TG($B_{p,l}$) with $3(p + l + 2)$ vertices and $4(p + l + 1)$ edges is disconnected. To make this as a connected graph include a new edge $b'b_1'$ to the edge set of $\beta'(G)$.Thus, we get an Extended triplicate graph of bistar with vertex set $\delta(G) = \delta'(G)$ and edge set $\beta(G) = \beta'(G) \cup b'b_1'$ denoted by ETG($B_{p,l}$). Clearly, ETG($B_{p,l}$) has $3(p + l + 2)$ vertices and $4(p + l) + 5$ edges.

3. MAIN RESULT

In this section, we investigate the existence of divisor cordial labeling, Sum divisor cordial labeling, Subtract divisor cordial labeling, Multiply divisor cordial labeling and Square divisor cordial labeling, in the context of Extended Triplicate graph of bistar.

THEOREM 3.1: Extended triplicate of bistar graph is a divisor cordial graph.

PROOF: The Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex set

$\delta(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_i \cup d_i' \cup d_i'' / 1 \leq i \leq p\}$ and edge set

$\beta(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_i' \cup b_1''d_i' \cup b_1'd_i \cup b_1'd_i'' / 1 \leq i \leq p\}$.

Clearly, It has $6(p + 1)$ vertices and $(8p + 5)$ edges.

To show that : ETG($B_{p,p}$) is a divisor cordial graph.

Define the function $S: \delta(G) \rightarrow \{1, 2, 3, \ldots.., p\}$ to label the vertices as follows.

$s(b) = 4(p) + 3$, $s(b') = 1$, $s(b'') = 4p + 5$,

$s(b_1) = 4(p+1)$, $s(b_1') = 2$, $s(b_1'') = 2(2p+3)$ and

For, $1 \le i \le p$	$s(c_i) = 2i+1$	$s(d_i) = 2(i+1)$
	$s(c_i') = 2(2p+i+3)$	$s(d_i') = 2(i+2p)+5$
	$s(c_i'') = 2i+2p+1$	$s(d_i'') = 2(i+1)+2p$

Define the function $S^*: \beta(G) \to \{0, 1\}$ is defined by

S*(bc) = $\begin{cases} 1 & (s(b)|s(c))\ or (s(c)|s(b)) \\ 0 & otherwise \end{cases}$, ∀ bc ∈ β(G) to label the edges as follows.

$$S^*(bb_1') = S^*(b''b_1') = 0 \text{ and } S^*(b'b_1) = S^*(b'b_1'') = S^*(b'b_1') = 1 .$$

For $1 \le i \le p$

$$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1'd_i) = S^*(b_1'd_i'') = 1 \text{ and}$$

$$S^*(bc_i') = S^*(b''c_i') = S^*(b_1d_i') = S^*(b_1''d_i') = 0 .$$

we get, $\beta_{s^*}(0) = 4p+2$ and $\beta_{s^*}(1) = 4p+3$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p+2) - (4p+3)| \le 1$

Thus, the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \le 1$ is satisfied.

Hence, Extended triplicate of bistar graph is divisor cordial graph.

EXAMPLE 3.1: Extended triplicate of bistar graph ETG($B_{3,3}$) and its divisor cordial labeling is shown in figure 3.

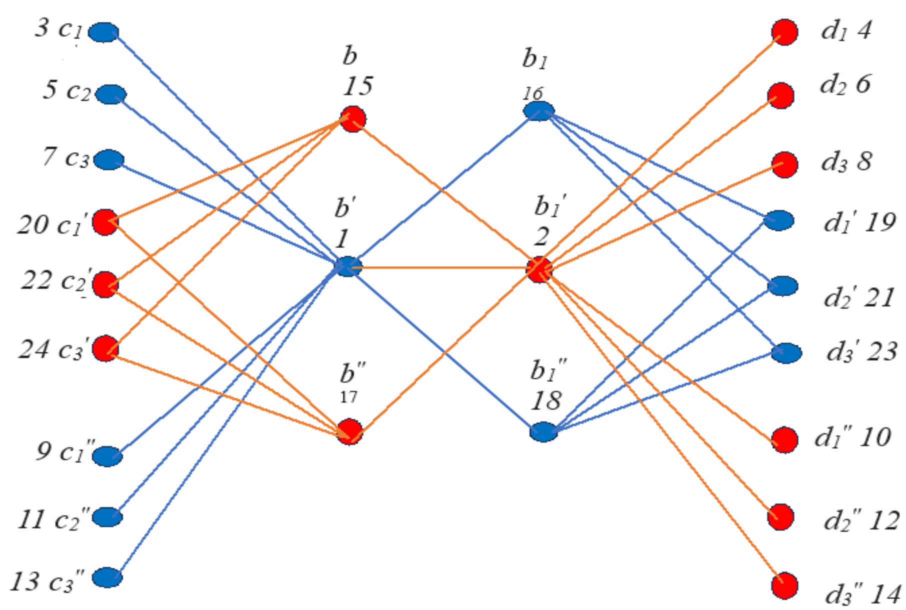

FIGURE – 1

THEOREM 3.2: Extended triplicate of bistar graph is a sum divisor cordial graph.

PROOF: The Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex set

$\delta(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_i \cup d_i' \cup d_i'' / 1 \leq i \leq p\}$

and edge set

$\beta(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_i' \cup b_1''d_i' \cup b_1'd_i \cup b_1'd_i'' /$
$1 \leq i \leq p\ \}$.

Clearly, It has $6(p+1)$ vertices and $(8p+5)$ edges.

lTo show that : ETG($B_{p,p}$) is a sum divisor cordial graph.

Define the function $S: \delta(G) \rightarrow \{1, 2, 3, \ldots., p\}$ to label the vertices as follows.

$s(b) = 3$, $s(b') = 2$, $s(b'') = 6$, $s(b_1) = 4$, $s(b_1') = 1$, $s(b_1'') = 5$ and

For, $1 \leq i \leq p$]	$s(c_i) = 2(i+3)$	$s(d_i) = 2i+5$
	$s(c_i') = 2(2p+i+3)$	$s(d_i') = 2(2p+i)+5$
	$s(c_i'') = 2(i+p)+5$	$s(d_i'') = 2(p+i+3)$

Define the function $S^*: \beta(G) \rightarrow \{0, 1\}$ is defined by $S^*(bc) = \begin{cases} 1 & ;\ if(2|s(b)+s(c)) \\ 0 & ;\ \quad otherwise \end{cases}$,
$\forall$ bc $\in \beta(G)$ to label the edges as follows.

$$S^*(bb_1') = S^*(b'b_1) = 1 \text{ and } S^*(b'b_1'') = S^*(b'b_1') = S^*(b''b_1') = 0 \ .$$

For $1 \leq i \leq p$

$$S^*(b'c_i) = S^*(b''c_i') = S^*(b_1'd_i) = S^*(b_1''d_i') = 1 \text{ and}$$
$$S^*(bc_i') = S^*(b'c_i'') = S^*(b_1d_i') = S^*(b_1'd_i'') = 0\ .$$

we get, $\beta_{S^*}(0) = 4p+3$ and $\beta_{S^*}(1) = 4p+2$

This implies, $|\beta_{S^*}(0) - \beta_{S^*}(1)| = |(4p+3)-(4p+2)| \leq 1$

Thus, the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate of bistar graph is sum divisor cordial graph.

EXAMPLE 3.2 : Extended triplicate of bistar graph ETG($B_{3,3}$) and its sum divisor cordial labeling is shown in figure 2.

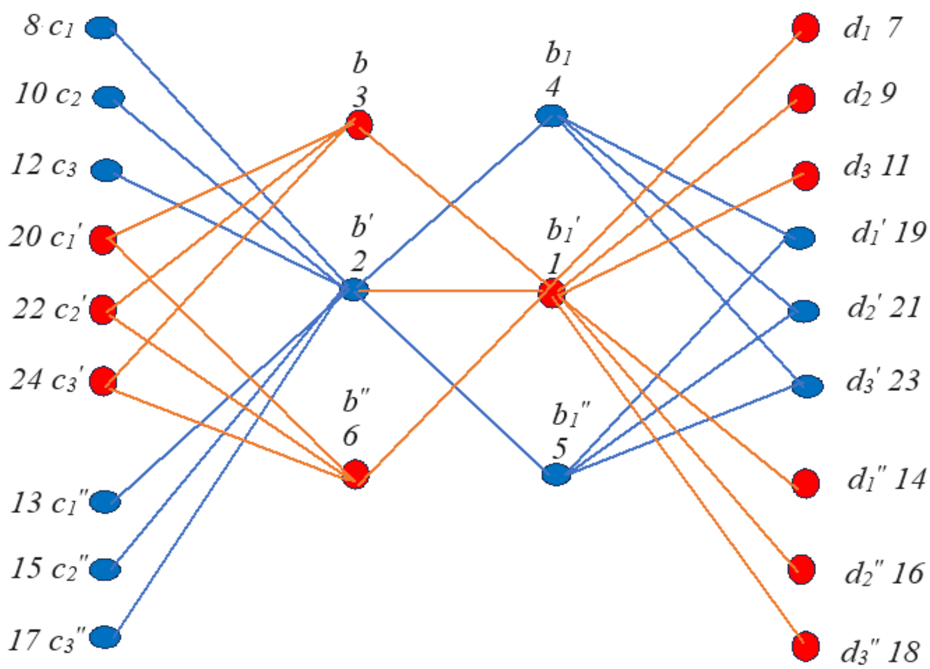

FIGURE -2

THEOREM 3.3: Extended triplicate of bistar graph is a subtract divisor cordial graph.

PROOF: The Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex set

$\delta(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_i \cup d_i' \cup d_i'' / 1 \leq i \leq p\}$ and edge set

$\beta(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_i' \cup b_1''d_i' \cup b_1'd_i \cup b_1'd_i'' / 1 \leq i \leq p\}$.

Clearly, It has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that : ETG($B_{p,p}$) is a subtract divisor cordial graph.

Define the function $S: \delta(G) \to \{1, 2, 3, \dots, p\}$ to label the vertices as follows.

$s(b) = 3$, $s(b') = 4$, $s(b'') = 6$, $s(b_1) = 2$, $s(b_1') = 1$, $s(b_1'') = 5$ and

For, $1 \leq i \leq p$	$s(c_i) = 2i + 5$	$s(d_i) = 2(i+3)$
	$s(c_i') = 2(i + 2p) + 5$	$s(d_i') = 2(i + 2p + 3)$
	$s(c_i'') = 2(i + 3 + p)$	$s(d_i'') = 2(i + p) + 5$

Define the function $S^*: \beta(G) \to \{0, 1\}$ is defined by

$S^*(bc) = \begin{cases} 1 \;; & if\ (2|s(b) - s(c)) \\ 0 \;; & otherwise \end{cases}$, $\forall$ bc $\in \beta$(G) to label the edges as follows.

$$S^*(bb_1') = S^*(b'b_1) = S^*(b'b_1'') = 1 \text{ and } S^*(b'b_1') = S^*(b''b_1') = 0 \;.$$

For $1 \leq i \leq p$

$$S^*(b'c_i) = S^*(b''c_i') = S^*(b_1'd_i) = S^*(b_1''d_i') = 0 \text{ and}$$

$$S^*(bc_i') = S^*(b'c_i'') = S^*(b_1d_i') = S^*(b_1'd_i'') = 1\,.$$

Here we get, $\beta_{s^*}(0) = 4p + 3$ and $\beta_{s^*}(1) = 4p + 2$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p + 3) - (4p + 2)| \leq 1$

Thus, the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate of bistar graph is subtract divisor cordial graph.

EXAMPLE 2.3 : Extended triplicate of bistar graph ETG($B_{3,3}$) and its subtract divisor cordial labeling is shown in figure 3.

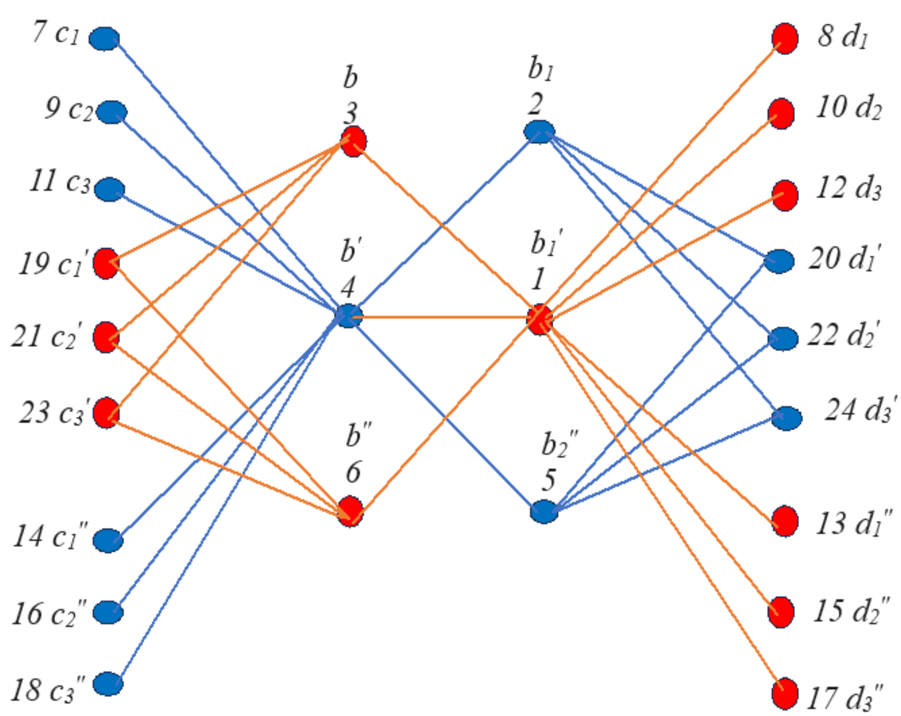

FIGURE – 3

THEOREM 3.4: Extended triplicate of bistar graph is a multiply divisor cordial graph.

PROOF: The Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex set

$\delta(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_i \cup d_i' \cup d_i'' / 1 \leq i \leq p\}$ and edge set

$\beta(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_i' \cup b_1''d_i' \cup b_1'd_i \cup b_1'd_i'' / 1 \leq i \leq p\}$.

Clearly, It has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that : ETG($B_{p,p}$) is a multiply divisor cordial graph.

Define the function $S: \delta(G) \rightarrow \{1, 2, 3, \ldots, p\}$ to label the vertices as follows.

$s(b) = 3$, $s(b') = 2$, $s(b'') = 5$, $s(b_1) = 4$, $s(b_1') = 1$, $s(b_1'') = 6$ and

For, $1 \leq i \leq p$	$s(c_i) = 2(i+3)$	$s(d_i) = 2i+5$
	$s(c_i') = 2(2p+i)+5$	$s(d_i') = 2(2p+i+3)$
	$s(c_i'') = 2(p+i+3)$	$s(d_i'') = 2(p+i)+5$

Define the function $S^*: \beta(G) \rightarrow \{0, 1\}$ is defined by $S^*(bc) = \begin{cases} 1 \ ; \ if \ (2|s(b).s(c)) \\ 0 \ ; \ \quad otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

$$S^*(bb_1') = S^*(b''b_1') = 0, \ S^*(b'b_1') = S^*(b'b_1'') = S^*(b'b_1) = 1 \text{ and}$$

For, $1 \leq i \leq p$

$$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1d_i') = S^*(b_1''d_i') = 1 \text{ and}$$

$$S^*(bc_i') = S^*(b''c_i') = S^*(b_1'd_i) = S^*(b_1'd_i'') = 0$$

Here we get, $\beta_{s^*}(0) = 4p+2$ and $\beta_{s^*}(1) = 4p+3$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p+2) - (4p+3)| \leq 1$

Thus, the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate of bistar graph is multiply divisor cordial graph.

EXAMPLE 3.4 : Extended triplicate of bistar graph ETG($B_{3,3}$) and its multiply divisor cordial labeling is shown in figure 4.

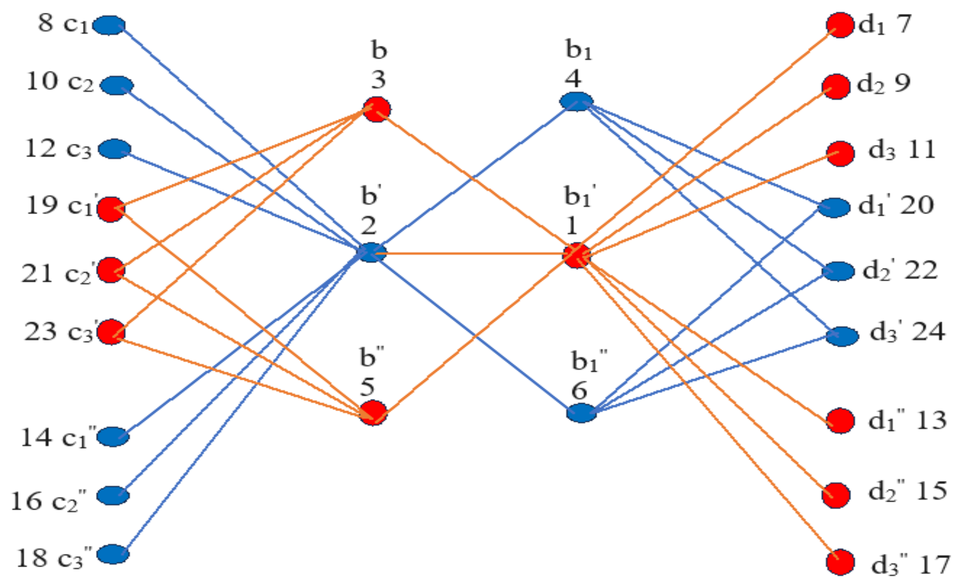

FIGURE – 4

THEOREM 3.5: Extended triplicate of bistar graph is a square divisor cordial graph.

PROOF: The Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex set

$\delta(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_i \cup d_i' \cup d_i'' / \, 1 \leq i \leq p \;\}$ and edge set

$\beta(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_i' \cup b_1''d_i' \cup b_1'd_i \cup b_1'd_i'' / \, 1 \leq i \leq p\,\}$.

Clearly, It has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that : ETG($B_{p,p}$) is a square divisor cordial graph.

Define the function $S: \delta(G) \rightarrow \{1, 2, 3, \dots, p\}$ to label the vertices as follows.

$s(b) = 4$, $s(b') = 2$, $s(b'') = 6$, $s(b_1) = 3$, $s(b_1') = 1$, $s(b_1'') = 5$ and

For , $1 \leq i \leq p$	$s(c_i) = 2i + 5$	$s(d_i) = 2(i+3)$
	$s(c_i') = 2(i + 2p + 3)$	$s(d_i') = 2(2p + i) + 5$
	$s(c_i'') = 2(i+p) + 5$	$s(d_i'') = 2(i + 3 + p)$

Define the function $S^*: \beta(G) \to \{0, 1\}$ is defined by $S^*(bc) = \begin{cases} 1\ ; & if((s(b))^2 | s(c)) or (s(b) | (s(c))^2)) \\ 0\ ; & otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to label the edges as follows.

we get, $\beta_{s^*}(0) = 4p + 3$ and $\beta_{s^*}(1) = 4p + 2$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p + 3) - (4p + 2)| \leq 1$

Thus, the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate of bistar graph is square divisor cordial graph.

EXAMPLE 2.5 : Extended triplicate of bistar graph ETG($B_{3,3}$) and its square divisor cordial labeling is shown in figure 5.

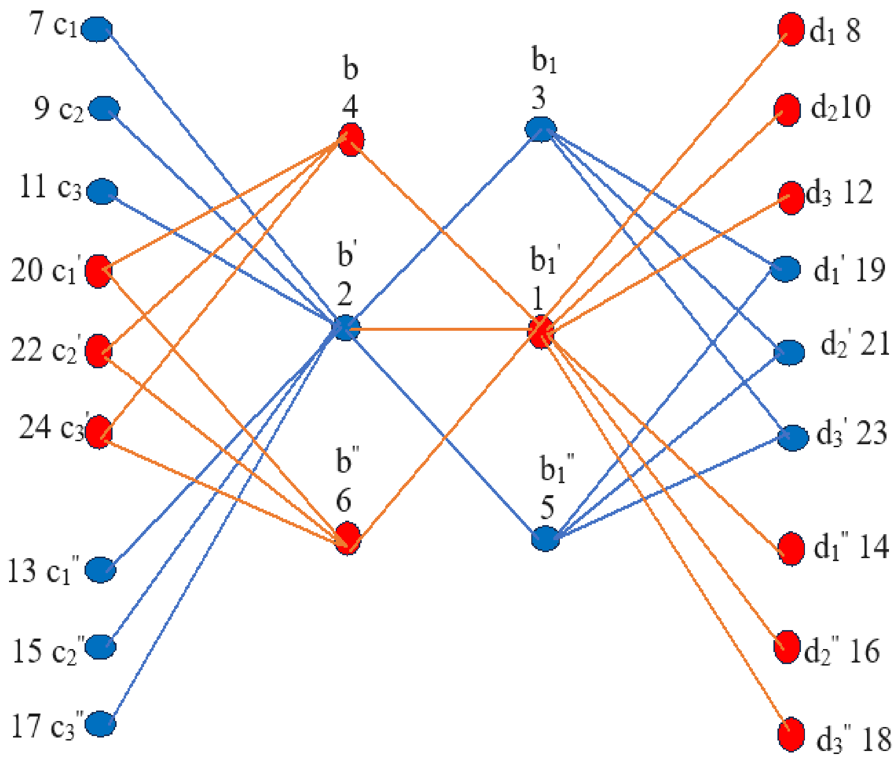

FIGURE – 5

CONCLUSION:

In this paper, we have investigated that the Extended triplicate of Bistar graph admits the Divisor cordial labeling, Sum divisor cordial labeling, subtract divisor cordial labeling, Multiply divisor cordial labeling and Square divisor cordial labeling.

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