Factorial Harmonious Labeling of Some Cycle Related Graphs

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Abstract

In this paper, we launch a new type of labeling said to be factorial harmonious labeling. Let G be a connected graph with m edges. A function f is called Factorial Harmonious Labeling of graph G if $f : V \rightarrow \{0, 1,2, \dots, 2m-1\}$ is injective and the induced function $f^* : E \rightarrow \{0,1,2, \dots, m-1\}$ defined as $f^*(e = ab) = \frac{[f(a)+ f(b)]!}{[f(a)]![f(b)]!} + \{f(a) + f(b)\}$ (mod m) is bijection. A graph which admits Factorial Harmonious labeling is called Factorial Harmonious graph. We discuss this labeling condition satisfies to path, bistar, butterfly $B_{3,n}$, (3,n)-kite graph.

Key Words: Factorial Harmonious labeling, Factorial Harmonious graph.

INTRODUCTION

A graph's vertex labeling G is a planning f made up of G's vertices to each edge ab has a label that depends on the vertices a and b and their label f(a) and f(b). A. Rosa [5] creates a Graph labeling methods in 1967. R. L. Graham *et al.* [4] proposed Harmonious graph notation in 1980 and A. Edward Samuel *et al.*[3] introduced the concept of Factorial labeling graph in 2018. We prove that the path, bistar, butterfly and (3,n)-kite graph are admits the factorial harmonious graphs.

KNOWN RESULT'S AND DEFINITION:

Definition: 1

Factorial Labeling was introduced by A. Edward Samuel and S. Kalaivani. A factorial labeling of a connected graph G is a bijection $f : V \to \{0, 1,2, \dots, m\}$ such that the induced function $f^* : E \to \{1,2, \dots, m\}$ defined as $f^*(e = ab) = \frac{[f(a)+ f(b)]!}{[f(a)]![f(b)]!}$ then the edges labels are distinct. Any graph which admits a **factorial labeling** is called a factorial graph.

Definition: 2

Harmonious labeling was introduced by R. L. Graham and N. J. A Sloane. Let G be a connected graph with m edges. A function *f* is called harmonious labeling of graph G if $f : V \to \{0, 1,2, \dots, m-1\}$ is injective and the induced function $f^* : E \to \{1,2, \dots, m\}$ defined as $f^*(e = ab) = \big(f(a) + f(b)\big)(mod\ m)$ is bijective. A graph which admits **harmonious labeling** is called harmonious graph.

Definition: 3

$B_{p,q}$ is the **bistar** obtained from two disjoint copies of $K_{1,n}$ by joining the centre vertices through an edge.

Definition: 4

A walk is called a **path** if all its vertices are distinct. A path on *n* vertices is denoted by P_n.

Definition: 5

Two cycles of the same order n sharing a common vertex with an arbitrary number m of pendant edges attached at the common vertex called **butterfly graph** $B_{n,m}$ where n.m are two positive integers.

Definition: 6

The **kite (m,n) graph** is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge.

FACTORIAL HARMONIOUS GRAPH

Definition:

Let G be a connected graph with m edges. A function f is called Factorial Harmonious Labeling of graph G if $f : V \to \{0, 1,2, \dots, 2m-1\}$ is injective and the induced function

$f^* : E \rightarrow \{0,1,2,\dots,m-1\}$ defined as $f^*(e = ab) = \frac{[f(a)+f(b)]!}{[f(a)]![f(b)]!} + \{f(a) + f(b)\}$ (mod m) is bijection. A graph which admits **Factorial Harmonious labeling** is called Factorial Harmonious graph and it is denoted by $Fl_{\mathcal{H}}(G)$.

Theorem: 1

Factorial Harmonious Labeling exists in the path graph P_n for all n .

Proof:

Let V(P_n) = $\{a_i : 1 \leq i \leq n\}$ and

E(P_n) = $\{ a_i a_{i+1} : 1 \leq i \leq n-1\}$

Then $|V(P_n)| = n$ and $|E(P_n)| = n - 1$

Define an one-one function f: V → {0,1,2,…, 2m-1}by

$f(a_i)$ = j , $0 \leq j \leq 2m-1; 1 \leq i \leq n$

The induced edge labels are

$f^*(a_i a_{i+1})$ ={ k , $1 \leq k \leq m; 1 \leq i \leq n-1\}$ (mod m)

f^*(E(G)) = { $0,1,2,\dots,m-1\}$

Then $|V[f(P_n)]| = 2m$ and $|E[f^*(P_n)]| = m$

Hence the path graph P_n admits Factorial Harmonious labeling for all n.

Illustration:

Consider the path graph P_3 is given in the following figure 3.2.5.

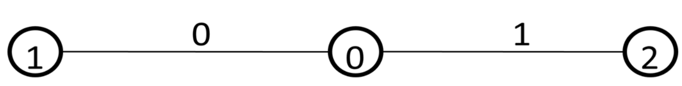

Figure 1

The path graph P_3 admits Factorial Harmonious labeling.

Theorem : 2

Factorial Harmonious Labeling exists in the bistar graph $B_{p,q}$ for all p < q .

Proof:

Let V($B_{p,q}$) =$\{a\} \cup \{b\} \cup \{a_i : 1 \leq i \leq p\} \cup \{b_j : 1 \leq j \leq q\}$ and

E($B_{p,q}$) = $\{ab\} \cup \{aa_i: 1 \leq i \leq p\} \cup \{ bb_j : 1 \leq j \leq q\}$

Then $|V(B_{p,q})| = p + q + 2$ and $|E(B_{p,q})| = p + q + 1$

Define an one-one function *f:* V → {0, 1, … , $2m - 1$} by

f(a) = 1

f(b) = 0

$f(a_i)$ ={ $j : 2 \leq j \leq 2m - 1$ and $1 \leq i \leq p$} ∪ $\{b_j : 1 \leq j \leq q\}$

The edge labels are as follows

$f^*(ab)$ = $\{\{k ; 1 \leq k \leq m\} \cup \{aa_i : 1 \leq i \leq p\} \cup \{bb_j: 1 \leq j \leq q\}\}$ (mod m)

$f^*(E(G))$ = $\{0,1,2, \dots , m - 1\}$

Then $|V[f(B_{p,q})]| = 2m$ and $|E[f^*(B_{p,q})]| = m$

Hence the bistar graph $B_{p,q}$ admits Factorial Harmonious labeling for all $p < q$.

Illustration:

Consider the bistar graph $B_{4,5}$ is given in the following figure 2.

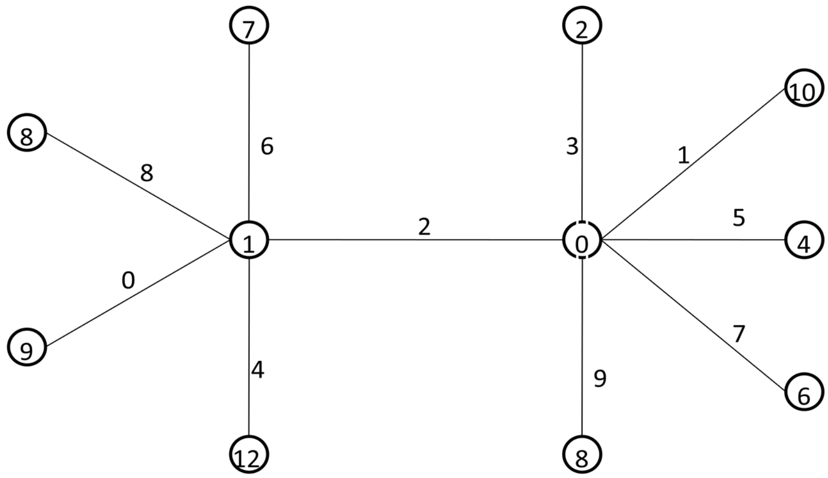

Figure 2

The bistar graph $B_{4,5}$ admits Factorial Harmonious labeling.

Theorem 3.3.4

Factorial Harmonious Labeling exists in the butterfly graph $B_{3,n}$ for all n .

Proof:

Let $B_{3,n}$ be the Butterfly graph.

Let V(G) = $\{a_1, a_2, a_3, a_4, a_5, b_1, b_2, \ldots, b_n\}$, let a_1, a_2, a_3, a_4, a_5 be the vertices of the two cycles C_3 and a_1 be the apex vertex of the two cycles C_3.

E(G) = $\{a_1a_2, a_1a_3, a_2a_3, a_1a_4, a_1a_5, a_4a_5\} \cup \{ a_1b_i : 1 \leq i \leq n \}$

Then $|V(B_{3,n})| = n + 5$ and $|E(B_{3,n})| = n + 6$

Define an one-one function f: V → $\{0, 1,2, \ldots 2m - 1\}$ by

$f(a_1) = 0$

$f(a_2) = 1$

$f(a_3) = 2$

$f(a_4) = 3$

$f(a_5) = 6$

$f(b_i) = j\ ,\ 4 \le j \le 2m-1;\ 1 \le i \le n$

The induced edge labels are

$f^*(a_1a_2) = 2;$ (mod m)

$f^*(a_1a_3) = 3;$ (mod m)

$f^*(a_2a_3) = 6;$ (mod m)

$f^*(a_1a_4) = 4;$ (mod m)

$f^*(a_1a_5) = 7;$ (mod m)

$f^*(a_4a_5) = 5;$ (mod m)

$f^*(a_1b_i) = k,\ 1 \le k\ \le m\ ;\ 1 \le i \le n$ (mod m)

f^*(E(G)) = $\{\ 0,1,2\ \dots, m-1\}$

Then $\left|V[f(B_{3,n})]\right| = 2m$ and $\left|E[f^*(B_{3,n})]\right| = m$

Hence the butterfly graph $B_{3,n}$ admits Factorial Harmonious labeling for any n.

Illustration:

Consider the butterfly graph $B_{3,2}$ is given in the following figure 3.

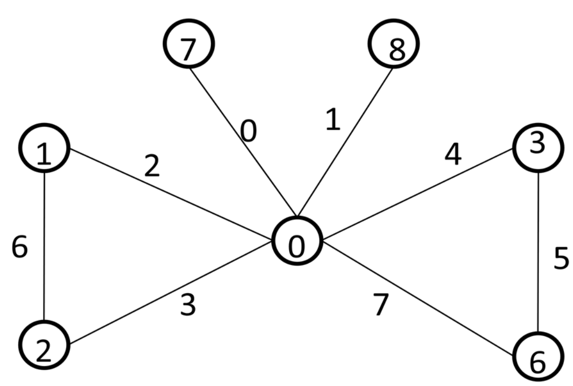

Figure 3

The butterfly graph $B_{3,2}$ admits Factorial Harmonious labeling.

Theorem: 4

Factorial Harmonious Labeling exists in the $(3, n\ )$ – kite graph for all n .

Proof:

Let $(3, n)$ be the Kite graph.

Let V(G) = $\{a_1, a_2\ , a_3, b_1, b_2, \ldots, b_n\}$, let $a_1, a_2\ , a_3$ be the vertices of the cycle C_3 and a_3 be the apex vertex of the path P_n.

E(G) = $\{a_1a_2, a_1a_3,\ a_2a_3\} \cup \{a_3b_1\}\ \cup \{\ b_ib_{i+1} : 1 \le i\ \ \le n - 1\ \}$

Then $|V((3, n)\ )| = n + 2$ and $|E((3, n)\ )| = n + 3$

Define an one-one function f: V → $\{0,\ 1,2, \ldots 2m - 1\}$ by

$f(a_1) = 2$

$f(a_2) = 3$

$f(a_3) = 1$

$f(b_i) = j\ ,\ 4 \le j \le 2m - 1;\ 1 \le i \le n - 1$

The induced edge labels are

$f^*(a_ia_{i+1}) = \{\{k,\ 1 \le k\ \ \le m\ ;\ \ 1 \le i \le n - 1\} \cup \{a_1a_3\} \cup \{a_3b_1\} \cup \{b_ib_{i+1} : 1 \le i \le n - 1\}\}\ (mod\ m)$

f^*(E(G)) = $\{\ 0,1,2 \ldots, m - 1\}$

Then $|V[f(3, n)]| = 2m$ and $|E[f^*(3, n)]| = m$

Hence the kite graph $(3, n)$ admits Factorial Harmonious labeling for any n.

Illustration:

Consider the kite graph $(3, n)$ is given in the following figure 4.

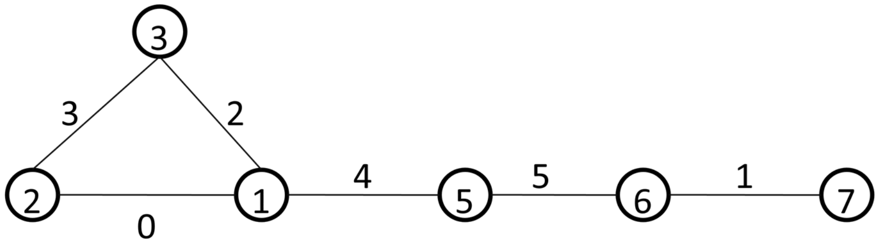

Figure 4

The kite graph $(3, n)$ admits Factorial Harmonious labeling.

Conclusion:

In this paper, we have shown that path, bistar, butterfly $B_{3,n}$ and (3,n)-kite graph are Factorial Harmonious labeling . In future the same process will be analyzed for other graphs.

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